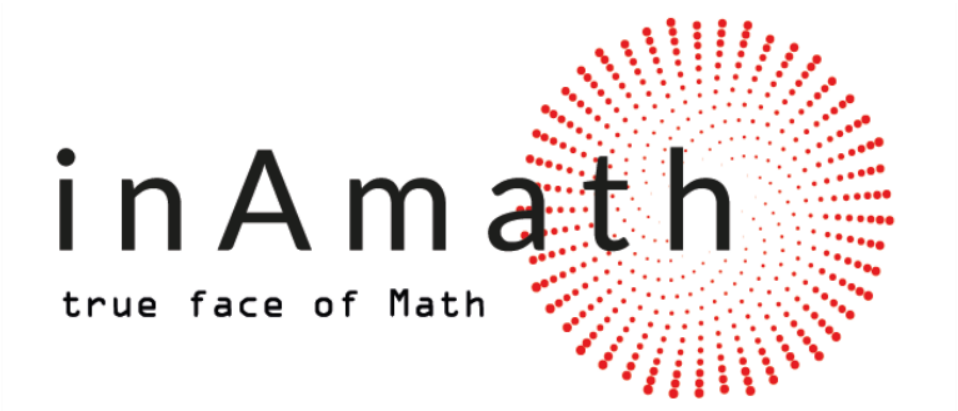


An interdisciplinary approach to mathematical education

SCENARIOS

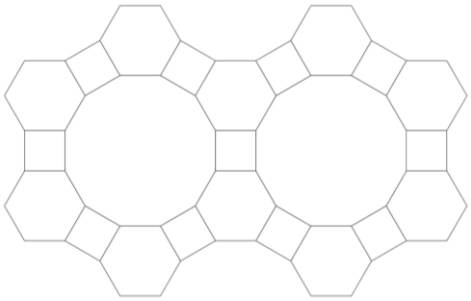
The content of scenarios is designed and created within the Erasmus+ project:

InAMath - An interdisciplinary approach to mathematical education

The project An interdisciplinary approach to mathematical education (2020-1-HR01-KA201-077816) is co-founded by the Erasmus+ Programme of the European Union, Key Action: Cooperation for innovation and the exchange of good practices - Strategic Partnerships for school education.

CONTENT

Title	**Count and colour me!**
Key words	straight and curved lines, squares and rectangles, colouring
Short description	As part of this activity, students use straight and curved lines to make tiles that pave the rectangle. In this task students compare the shapes of the tiles, and special emphasis is placed on recognizing and distinguishing squares and rectangles. The created tiles are coloured according to the instructions that change from task to task, thus practicing the skill of colouring and developing and applying strategies for colouring according to given instructions. In the final part of the activity, students make and use a tangram puzzle. It is possible to continue the activity in IT classes.
IT tools	
Fields (select)	**A1: Mathematics** A2: Natural science **A3: Art** A4: Music A5: PE A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: straight and curved lines; squares and rectangles; counting, comparing
	A3: colouring, cutting
Expected prior knowledge	straight line, curved line, square, rectangle
Expected learning outcomes	• Uses mathematical reasoning and mathematical language to display and solve different types of tasks. • It singles out and names geometric bodies and figures and connects them with the shapes of objects in the environment. • Draws and distinguishes straight and curved lines. • Analyses and compares objects from the environment according to a measurable property. • The student demonstrates knowledge of the peculiarities of various visual materials and procedures in artistic expression.
Expected duration	90 minutes
Preparation	1. Prepare worksheets for each student (https://inamath.uniri.hr/wp-content/uploads/2022/11/Count-and-colour-me-worksheet-v2.docx) 2. Math reading: The story of the tangram (https://inamath.uniri.hr/wp-content/uploads/2022/11/The-story-of-tangram.docx)
Detailed description of	Worksheets with the following tasks are distributed to the students.

activities	Note: the first two tasks can be given to students for independent work that precedes classroom activity. 1. Colour the picture below by colouring the tiles that are the same shape using the same colour.  Students write the colours they used and the number of tiles they painted with that colour in the table (sheet). Together with the teacher, students describe the three different shapes of tiles that appear and note which shape appears the most. 2. Divide the rectangle into multiple tiles using 6 curved lines.  Students write the number of tiles in the table. If desired, students colour the tiles however they want. 3. Divide the rectangle into 10 tiles using straight lines and colour them so that all tiles are different colours.  Note: this task is already a problem for students because they have to control the number of parts. That is why it is desirable that the teacher, before the students start drawing independently, make a division on the board. Also, the teacher should use tiles of different shapes, only some of which are rectangular. This is an opportunity to describe a rectangle and distinguish it from another quadrilateral. 4. Divide the rectangle into multiple rectangle-shaped tiles.

Students write the number of rectangular tiles in the worksheet. Students are asked: Is there a square between them? With the students, the square is described and the differences between a square and a rectangle are noted, that is, it is concluded that a square is a special case of a rectangle.

Furthermore, students are given the following task:

Colour each tile in one colour, but so that no two adjacent tiles are the same colour, noting that adjacent areas are those that touch lengthwise.

5. Draw straight lines so that you have at least 2 tiles in the shape of a square.

Students analyse the obtained picture with the help of the following questions: Which tiles are there more of, rectangular that are not square or rectangular that are square?

Furthermore, the colouring process becomes more difficult; students should use as few different colours as possible. They are instructed to do so:

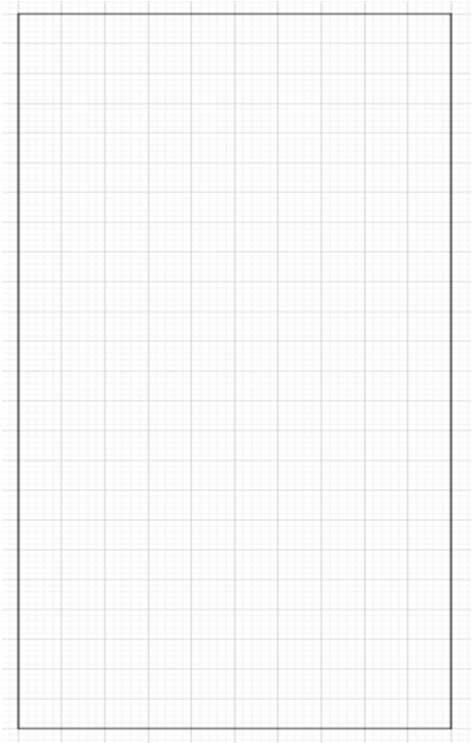
Colour each tile so that every two adjacent tiles are coloured differently. Try to use as few different colours as possible. How many colours did you use?

At this moment, the teacher tells the students that mathematicians have shown that they could paint the tiles with a maximum of 4 colours and explains the strategy how they can achieve this using the example that the teacher drew on the board in task 3. The teacher takes one colour and paints one tile. Students recognize which tiles the teacher is no longer allowed to paint with that colour, that is, which tiles he can paint. After the teacher paints the second tile using the first colour, they decide together again which tiles should not be that colour. After there are no more tiles teacher can paint using the first colour, the teacher takes another colour and repeats the same process...

6. In this task, students divide a rectangle without any special instructions, but they have an instruction for the number

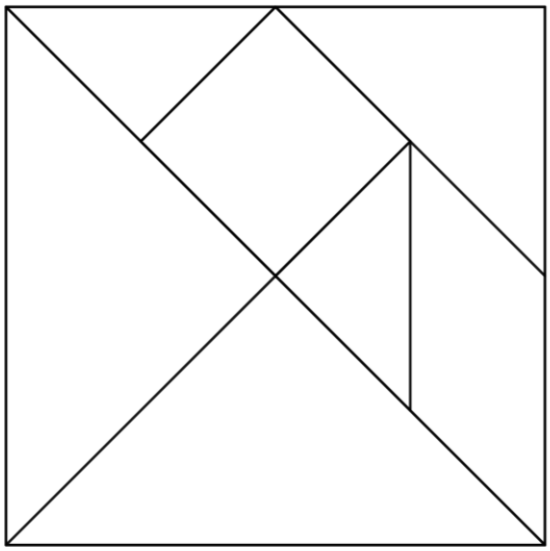
of colours.

Divide the rectangle into as many and whatever tiles you want. You can use both straight and curved lines.

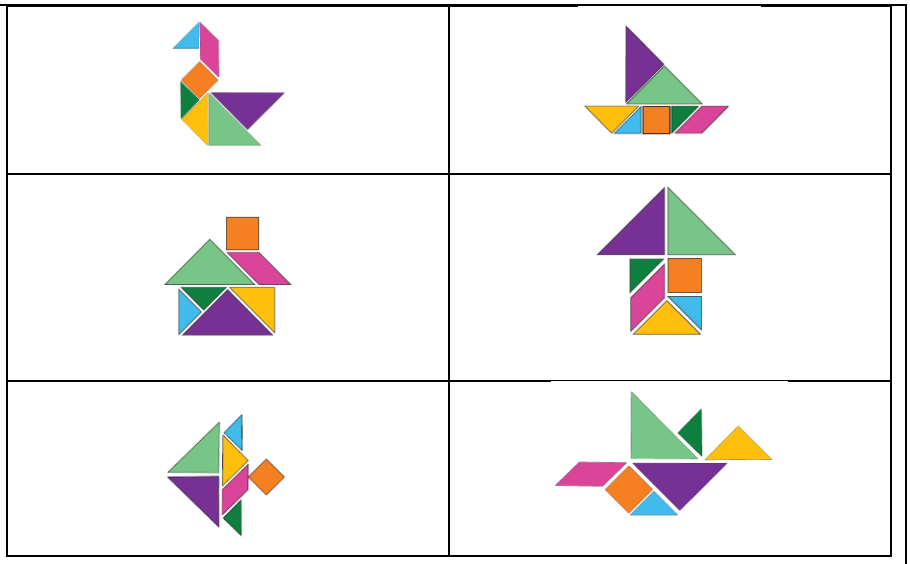


Students are instructed to use 4 or fewer colours (so that every two adjacent tiles are different colour).

7. Colour the tiles in the next square.

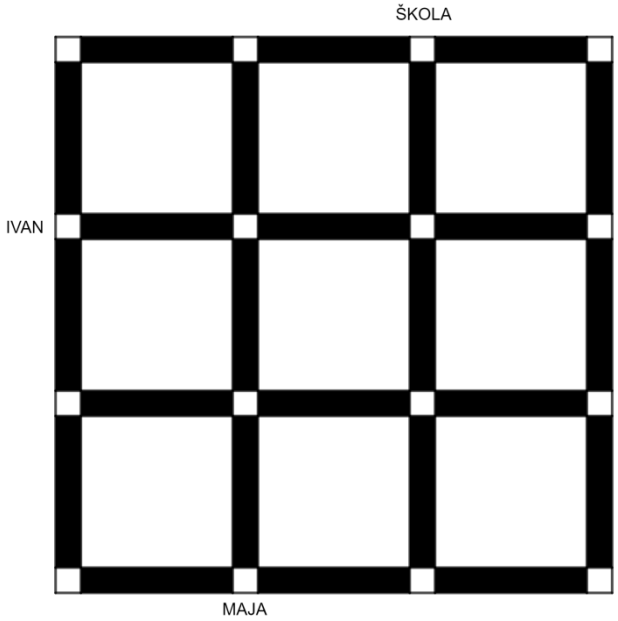


Cut out the tiles and try to make some of the following tangram shapes with them.

	 Note: the last task can be given to the students for homework and the teacher can read them the Tangram Story.
Extension activities	In IT class (in the same or higher grades), students can draw different "tiles" on the computer (eg pentomino shapes, trangram shapes, tetris shapes, etc.) and make different shapes from them on the computer. Students can use different software tools to draw tiles: GeoGebra, Scratch, Logo, Tinkcercad. In addition, students can prepare "tiles" that will be made with the help of a 3d printer or a laser cutter.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Math ride**
Key words	neighbourhood; traffic; navigation in space; counting and calculating to 20
Short description	In this activity, students link mathematics with natural sciences and create their own city in which they move and find their way. During the activity, pupils independently analyse a given building plan and, according to it, create their own neighbourhood and set up additional facilities. By solving problem tasks, apart from achieving math learning outcomes, pupils develop the skills of visualisation and orientation in space. The activity can be extended in IT class.
IT tools	Maqueen micro:bit; micro:bit
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: PE A6: Mother tongue A7: Foreign language Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: calculating to 20
	A2: city area, neighbourhoods; cities, traffic, traffic signs, distance, navigation in space
Expected prior knowledge	counting to 20; orientation in space; calculating to 20; traffic; traffic signs
Expected learning outcomes	• Describes and displays quantities by natural numbers and zero. • Adds and subtracts in a set of numbers up to 20. • Uses mathematical reasoning and mathematical language to display and solve different types of tasks. • It sets up a mathematical problem (determines what is known and unknown, predicts/researches and selects strategies, draws conclusions and determines possible solutions). • It uses data and displays it with pictograms and simple tables. • The student compares the organization of different spaces and communities in the immediate environment.
Expected duration	90 minutes
Preparation	The following should be prepared: 1. Parts of the road that the students will assemble from paper or printed on a 3D printer and a plan of one city area (GeoGebra road: https://inamath.uniri.hr/wp-content/uploads/2022/11/voznja_-ravno.ggb , https://inamath.uniri.hr/wp-content/uploads/2022/11/voznja_T.ggb , https://inamath.uniri.hr/wp-content/uploads/2022/11/voznja.ggb , https://inamath.uniri.hr/wp-content/uploads/2022/11/voznja_kut.ggb; 3d print: https://www.tinkercad.com/things/8ayryvLRFLL).

	Note: as preparation for the activities, students can independently draw and colour parts of the road. 2. Traffic signs: stand and sign printed on a 3d printer or made of cardboard and/or sticks (https://www.tinkercad.com/things/b50dEyh9JGw); stickers with signs Note: As preparation for the activities, if possible, students can draw the signs themselves. https://www.instruktor-voznje.com.hr/prometni_znakovi/ 3. Buildings, people and cars (toys) Note: As a preparation for the activity, students can only make buildings in the form of different geometric bodies from cardboard. They can also make cardboard people. 4. If possible, it is preferable to have at least one maqueen car that is programmed for driving in the Ideal city are available at: https://inamath.uniri.hr/math-ride/ 5. Residential area and neighbourhood plan (https://inamath.uniri.hr/wp-content/uploads/2022/11/city-map.docx) 6. Worksheets (https://inamath.uniri.hr/wp-content/uploads/2022/11/A-mathematical-tour-worksheets.docx) The activity is carried out as group work in 4 groups. Each group makes its own neighbourhood. A neighbourhood has at least: two buildings, two people, two to three signs, 24 straight sections of road and 16 intersections. On top of that, it is necessary to have at least a dozen more straight sections and intersections to connect the districts into one city.
Detailed description of activities	1. MOTIVATION (5 minutes) Students repeat the characteristics of urban settlements and cities and list everything that exists in cities. Furthermore, the teacher asks the students if they know why it is important to plan cities well, especially traffic in cities, and announces that today the students will make their own mathematical city in which it will be easy to navigate and comfortable to live in. 2. MAKE YOUR NEIGHBOURHOOD (20 min) For the purposes of this activity, it is desirable to provide space for stacking the city on the floor. Each district requires approximately a square-shaped space with a side length of 1 m. Each group of students arranges their neighbourhood, and for this they need the following: streets, road signs, buildings, people, and vehicles. Each group of students chooses one student, the leader, who oversees all phases. • STREETS Each group of pupils receives a neighbourhood plan and parts with which they can build their road (straight parts and intersections).



- TRAFFIC SIGNS

Each group of students gets two to three stands, a dozen signs and paper with stickers and the task of making three signs that they know the meaning of and placing them somewhere in their neighbourhood.

- BUILDINGS

  Each group gets one building, a school, which they place as indicated in the neighbourhood plan.

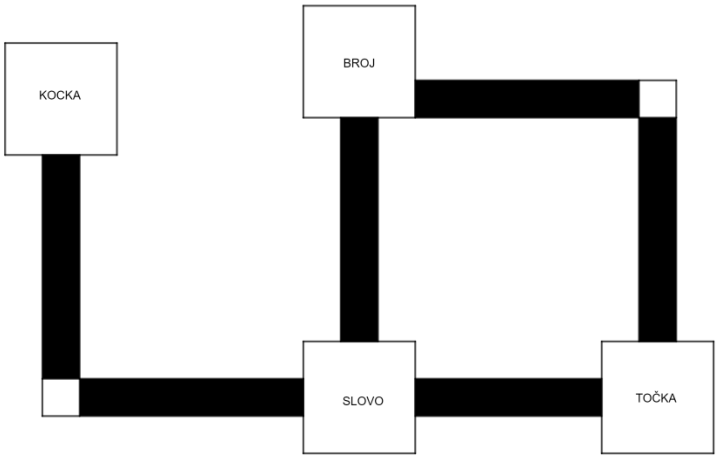
- PEOPLE

  Each group gets two people that they place as indicated in the neighbourhood plan. The teacher should emphasize to the students in which way the persons are turned (orientated). They must be oriented so that they can go straight down the road and turn left or right.

- VEHICLES

  Each group gets two vehicles to drive around their neighbourhood.

3. BUILD YOUR TOWN (10 minutes)
   In the end neighbourhoods are merged into Matkograd according to the given plan. Group leaders participate in the merger.



4. WHAT WOULD IT BE LIKE TO LIVE IN A MATH CITY? (30 minutes)
   Students are given worksheets on which they have tasks to solve by observing the neighbourhoods they have arranged.
5. WE DRIVE THROUGH AN IDEAL CITY (15 minutes)

	The group that first solves the last task in the worksheet drives the maqueen along the path they counted the steps. Driving a maqueen car is usually interesting for students and everyone would like to try it. If possible, we advise you to ensure enough time for this part of the activity and as many cars as possible.
Extension activities	1. Using prepared materials, similar activities can be carried out in higher grades with more complex city plans and more complex tasks. For example, when teaching the calculation of the length, when teaching the measurement units for length and conversion, when teaching the calculation of the area, etc. 2. In IT class students can create or paint parts of the road on the computer (eg. in Paint, GeoGebra, Tinkercad, etc.). 3. In higher grades, students can programme the maqueen, a 3d road model or draw a road in GeoGebra and model signs in Tinkercad.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Drawing with symmetries**
Key words	writing and recognizing numbers and letters; axial symmetry; drawing with watercolours and tempera paint; memory game
Short description	As part of this activity, students are introduced to the concept of symmetry, i.e. axisymmetric shapes, and learn how to recognize and draw such shapes. Analysing images, students recognize the letters, numbers, symbols and figures they have met and notice the properties of individual shapes and the differences between shapes. By drawing axisymmetric shapes, students develop the skill of drawing with watercolours or tempera. It is possible to continue the activity in IT class.
IT tools	
Fields (select)	A1: **Math** A2: **Natural science** A3: **Art** A4: Music A5: PE A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: Writing numbers and math symbols; drawing plane shapes; counting to 20
	A2: recognition of objects, beings and phenomena from the environment
	A3: drawing with watercolours and temperas, cutting
	A6: writing and letter recognition
Expected prior knowledge of pupils	Students know letters, numbers, mathematical symbols and basic plane shapes.
Expected learning outcomes	• Describes and displays quantities by natural numbers and zero. • Compares natural numbers up to 20 and zero. • Uses mathematical reasoning and mathematical language to display and solve different types of tasks. • It singles out and names geometric solids and figures and connects them with the shapes of objects in the environment. • Analyses and compares objects from the environment according to a measurable property. • The student compares organization in nature by perceiving the immediate environment. • The student recognizes and interprets in his artistic work the connection of shaping the visual environment with the activities, contents and purposes that take place in it. • The student uses the visual language in the creative process and expression so that he starts from the experience of the whole to the detail. • The difference between the character and the body.

	Relationships: larger, smaller, equal on the surface and in space; adding and subtracting forms. • The student uses some of the proposed art materials and techniques • The student reads texts appropriate to initial literacy and features of language development. • The student writes in the school formal letters, words and short sentences in accordance with language developments.
Expected duration of the activity	90 minutes
Preparation	Prepare cards with letters, numbers, plane shapes, symbols, etc. (https://inamath.uniri.hr/wp-content/uploads/2022/05/Kartice.pdf) Prepare square-shaped cardboard for memory cards and paper of the same shape.
Detailed description of activities	6. PART 1 (15 min): Introducing the concept of axial symmetry and observing axisymmetric numbers and letters Have you ever heard of the term symmetry? Today we are going to talk about a concept called axial symmetry. We will say that a plane shape is axisymmetric if we can draw it by folding paper. Students (in pairs) are given cards with letters, numbers, and plane shapes. Students select cards with axisymmetric plane shapes and fold the cards along the axis of symmetry. 7. PART 2 (45 min): Developing the technique of drawing symmetrical shapes by folding paper and "mirroring" one part of the shape on another part of the paper Each student gets an A4 paper that they fold in half. On one half using a pencil, students draw half a tree, leaves and apples on the tree... Painting green, brown and red tempera or watercolours over the pencil sketch, and by folding the paper, the drawn part is copied onto the other part of the paper. We comment with how we drew an axisymmetric image. 8. PART 3 (30 min): Designing a memory game and making memory cards using the paper folding technique. We distribute cardboard cards and pieces of paper of the same dimensions as the cardboard to the students. Using the paper folding technique, we make pairs for the memory game: on one card is an axisymmetric plane shapes, and on the other is a description of that shape (text). Students are given the task to make, for example, the following pairs: • A - the first letter of the alphabet • 8 - eight • Image of a circle - Circle (text) • Image of a square - Square (text) • Image a rectangle - Rectangle (text) • Image of the sign = - Equal (text)

	• Image of the sign > - Greater (text) • 0 – Zero (text) • Image of a swallow - Swallow (text) • Image of acorn - Acorn (text) • Image of an apple - Apple (text) • Image of a leaf - Leaf (text) • Image of a heart - Heart (text) • Image of a butterfly - Butterfly (text) Students draw images from the above pairs on a piece of paper in the following way: fold the piece of paper in half, draw one half of the image on half of the paper, and by folding the paper the other half is outlined. Stick a piece of paper with an image of an axisymmetric figure on cardboard. Then, on a new piece of cardboard, they write the text that corresponds to the picture they drew. This makes one pair for the memory game. Each pair of students makes their own game (out of 14 pairs), so that each student makes one pair at school and the remaining pairs for homework. Students bring the created cards to school and play the game.
Extension activities	4. Symmetrical shapes can be recognized and drawn with the help of a computer, for example using the GeoGebra program. 5. Making Christmas/New Year decorations using the paper folding technique. 6. Fold the square paper in half. Cut the folded paper into the desired shape and cut holes in it. After cutting, we "open" the paper and we got a pine flake/ball that is axisymmetric. In this way, effective paper snowflakes can be made. 7. Creation of the game Spot it!
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Draw my shadow**
Key words	Solid shapes and plane shapes, shadow of objects
Short description	As part of this activity, students get to know the shadows of certain solid and plane shapes by matching the shadows of geometric shapes with the shadows they have seen outside the classroom. Analysing a geometric shape and its shadow, students notice the properties of individual shapes and the differences between shapes, with special emphasis on the difference between plane shapes and solids and the differences between a square and a cube. Students make wire models of the body and in this way observe their properties and acquire related concepts. By drawing the shadows of the body, students develop the skill of spatial drawing using a pencil. It is possible to continue the activity in IT class.
IT tools	
Fields (select)	A1: **Math** A2: **Natural science** A3: **Art** A4: Music A5: PE A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: solid shapes, plane shapes, numbers up to twenty
	A2: shadow and how shadow is created
	A3: drawing and colouring, making the body
Expected prior knowledge	Plane shapes, solid shapes (the activity can be carried out with students who have already learned solids, but also as motivation when introducing solid shapes)
Expected learning outcomes	• It singles out and names geometric solids and figures and connects them with the shapes of objects in the environment. • Uses mathematical reasoning and mathematical language to display and solve different types of tasks. • Compares natural numbers up to 20 and zero. • The student navigates in the space around him by respecting the rules and concludes about the impact of the change of position on relationships in the space. • The student recognizes art as a way of communication and responds to various incentives with artistic expression. • The student demonstrates knowledge of the peculiarities of various visual materials and procedures in artistic expression.
Expected duration	90 minutes
Preparation	1. Wire models: triangle, square, rectangle, pentagon, cube, cuboid, three-sided pyramid, four-sided pyramid and optionally cone and cylinder. 2. A cube-shaped box without one side, with one side (opposite the hole) being a rectangle made of light-transmitting

	material (eg greaseproof baking paper). The teacher places shapes inside the box that he illuminates while the students observe the shadow of that body. 3. A lamp with which we will make the shadow, for example the light on a mobile phone, an LED lamp or natural daylight. 4. Students make wire models of the bodies, tops from balls of cork or similar and edges from toothpicks, sharpened sticks for skewers, matches, etc. It is preferable to use different colours of sticks or tops so that it is easier to count. If a cork is used, one cork per student is enough, which will be cut into approximately 16 equal-sized pieces, and it is necessary to prepare enough thin sticks that are sharpened on both sides.
Detailed description of activities	9. WHICH ITEM AM I HOLDING IN MY HAND? (20 minutes) The teacher shows the shadows of triangles, squares, and rectangles to the students and the students have to recognize what the teacher is holding in his hand and how big that object is, whether it is as big as its shadow or bigger or smaller. The teacher moves the light source farther and closer and the students conclude what the size of the shadow depends on. Furthermore, the teacher asks the students if the shadow will always have the same shape as the plane shape. This part describes what a shadow is and how it is created. Students are reminded that they too have a shadow and are asked if their shadow is always the same and if they know what it depends on. At the end of the introductory part, the question arises as to what the shadows of the solid shape look like. Students list the solids they have met so far, and the difference between round and angular bodies is introduced. Next, an activity for angular solids is carried out. 10. MAKE A MODEL AND LOOK AT MY SHADOW (60 min) Students work in pairs. CUBOID 8 balls and 12 sticks (8 of one length and 4 of another length) The teacher shows the cuboid model he has prepared and the students count the vertices, edges and sides and compare the lengths of the edges. From the sticks and balls on their desk, students choose the ones they need and make a wire model of a cube. The teacher shows the shadow of the cuboid that the students are trying to draw on paper. If they fail, the teacher helps them by making a shadow over the paper and the students cross out the peaks and shadows that fell on the paper. Together with the students, the teacher notices that the shadow of the solid shape is composed of plane shapes. They observe that the total number of vertices and edges on the shadow is equal to the total number of vertices and edges of the solid shape. Furthermore, the total number of faces is equal to the number of plane shapes in the shadow, but the faces and plane shapes are not of the same shape. Below, the same activity is carried out for a cube and a three-sided and four-sided pyramid.

	• DICE: 8 balls and 12 sticks of equal length • PYRAMID (THREE SIDED): 4 balls and 6 sticks • PYRAMID (FOUR-SIDED): 5 balls and 8 sticks
Extension activities	8. Students can make solid shapes they are not familiar with and explore their shadows; moreover, students can recognize solids from shadows and highlight their properties. 9. As part of art class, students can paint shadows and decorate them in different ways. 10. In IT class, students can create, observe and analyse individual bodies in a 3d browser or software tool for 3d graphics (eg GeoGebra, Tinkercad).
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Math card trick: Which card is it?**
Key words	math trick with cards, counting to 20, digit one and digit ten
Short description	The activity is based on mathematical tricks with cards which develops a positive attitude towards maths in students. This activity is an excellent way to strengthen pre-mathematical and mathematical skills in a fun and creative way that increases students' interest and success. The math background of the "Which card is that?" trick involves counting to 20 and recognizing digits one and ten. The activity develops students' motor skills, the ability to follow and reproduce the procedure and the focused implementation of the procedure according to exactly given instructions, in which the final success depends on the successful implementation of each individual step. It is possible to extend the activity by creatively designing performances within mother tongue, foreign language, art, and music classes.
IT tools	
Fields (select)	A1: **Math** A2: Natural science A3: Art A4: Music A5: **PE** A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: pre-math and math skills: repeating the procedure in the exact order; digit one and digit ten; counting to 20
	A5: motor skills and fine motor skills
	A6: creating a story; communication with the audience; giving clear instructions
Expected prior knowledge	counting to 20, one and ten digits
Expected learning outcomes	• Describes and displays quantities by natural numbers and zero • Use ordinal numbers up to 20. • Adds and subtracts in a set of numbers up to 20. • Follow the operating instructions and rules of motor play • The student uses words, phrases and sentences in the correct sense in normal communication situations • The student talks and speaks in accordance with language development expressing his needs, thoughts and feelings. • The student creatively expresses himself according to his own interest motivated by different experiences and experiences of literary text
Expected duration	90 minutes
Preparation	1. Prepare decks of cards (one deck per two students)

	2. The teacher practices the trick 3. The teacher learns the math behind the trick as instructed (https://inamath.uniri.hr/wp-content/uploads/2022/11/Which-card-is-it-instructions.docx) 4. Worksheet - thought experiment (https://inamath.uniri.hr/wp-content/uploads/2022/11/Which-card-is-it-worksheet.docx)
Detailed description of activities	PART 1: Presentation of the trick by the teacher (15 minutes) The teacher announces that a math card trick will be performed. To perform the trick, it is not necessary to have dexterous hands, but to let the math do the magic part. At the beginning of the presentation, the teacher explains what a math card trick means: in math card tricks, the trick relies exclusively on math, without the necessary skills of the trick performer, without "fake" and marked cards and hidden information. In the introductory part, it is emphasized that the most important part of today's activity is the following. • Students listen carefully to the teacher and follow the instructions. • Students carefully, slowly and concentrated carry out everything the teacher tells them. • Students do not play with the cards but stop after each individual step and wait for further instructions. It is important that the teacher and students are positioned so that all students can see the performance of the trick as much as possible, for example, if the teacher sits on the floor and the students gather around. To perform this trick, the teacher needs to separate more than 20 cards from the entire deck of cards (for example, he can tell the students that he will separate as many cards as there are students in the class, if there are more than 20, or say: we will separate 25 cards). The teacher performs the trick several times as instructed without further explanation. Students observe the trick. This part of the activity usually results in students' enthusiasm, disbelief, and questions like "How did you do that?" The teacher asks the students if they want to know how and why the trick works and if they want to learn how to do the trick. PART 2: Disclosure and explanation of the trick (45 minutes) In this step, the teacher explains why the trick works, that is, the math behind the trick. For math tricks, this is a key part because the goal is for students to understand the math background of the trick, i.e., why it works, because only then will they think that the trick is not based on fraud but on mathematics, i.e., science, and that there is a logical explanation why the trick „works". The teacher gives the students a worksheet (thought experiment) that the students fill in independently (each student individually). When the students complete the first part, the teacher "guesses" that all the students got a 9 as the result. The enthusiasm of the students is expected. At this moment, the teacher emphasizes the math behind the trick: for every number between 11 and 19, the following applies: when we subtract the sum of its digits from that number, the result is 9. The students are given the task of proving this, i.e., that of every number

	greater than 10 and less than 20; subtract the sum of its digits (second part of the worksheet). Now let us go back to the card trick and explain where we used this procedure in performing the trick. Suppose that the student (assistant) at the moment when the teacher (performer of the trick) asked for a number greater than 10 and less than 20 said the number 14. Here we explain to the students that we need more than 20 cards to perform the trick because in the "worst" case the assistant can say the number 19, so we need to separate 19 cards from the whole pile of cards that we have counted. The teacher reminds the students: then we separate 14 cards. Then, of those 14 cards, we move 5 of them. We have 9 cards left in our hand. We give the card from the top to someone. As that card is the first card on top of the stack of cards in our hand, and we have a total of 9 cards in our hand, it means that if we turn the whole stack of cards face up, that card is ninth in order. We ask the students if we had a chance to see the ninth card when performing the trick. Students are expected to remember the beginning of the trick and count the cards face up. Let us emphasize that we memorized the ninth card then. To make sure the students understand the math behind the trick, we repeat the trick once more, but this time we emphasize when we get to the ninth card (at the beginning of the trick, when counting cards face up) and turn the ninth card upside down (opposite to all the other cards). We turn over the entire pile of counted cards so that all cards are now face down (except for the ninth card). We emphasize to the students that this card is still ninth in order (from the top of the pile in our hand). We show it to the students. Then we separate the 14 cards one by one. We emphasize to the students that when separating the cards, we change the order of the cards one by one, i.e., after we have counted 14 cards, the face-up card is no longer the ninth in order from the top of the pile in our hand, but from the bottom. We still have to remove 5 cards from those 14 cards. After moving, we have exactly 9 cards left in our hand, and we see that on top of the pile of cards left in our hand will be exactly the card we memorized/turned over at the beginning of the trick. We emphasize once again that no matter what number the assistant tells us, we will always have nine cards left in our hand (which we have shown in the worksheet), so this means that we will always get to the "ninth/faced card". PART 3: Practicing a trick (30 minutes) Now the activity is transferred to the students, and they try to repeat the trick, carrying out the procedure that the teacher showed them. Students, working in pairs, repeat the trick until they practice it and learn to perform it without error. It is essential that at least once all pairs of students perform the trick simultaneously (all pairs for the same number), following the teacher's instructions, and that the student controls the implementation. During the implementation, the teacher repeats the trick one more time and the students make sure that the explanation is correct by counting the cards

	and following the ninth card. At the beginning, students can perform the trick with the ninth card face up, and when they are confident, they perform the trick without turning over the ninth card at the beginning. When practicing the trick, the students are emphasized to try to tell a "story" along with the trick: figure out why we set aside, for example, 25 cards at the beginning, and to practice the story at the end of the trick: revealing information piece by piece. For homework, students have to practice the trick and present it to their housemates (students are given written instructions for performing the trick).
Extension activities	1. Create the story and details that will make the trick interesting for the audience (e.g., students create and rehearse the scenario for part of the trick when they "guess" the card that everyone has seen). 2. Designing and/or finding music and scenery to perform the trick. 3. Rehearsal and performance. Each trick is a small performance that students must practice and perform in front of an audience. The trick requires the students to talk to the audience, give clear instructions, and lead the audience through the story they have created so that the trick is more than just a mathematical procedure.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Geometric shapes**
Key words	basic geometric shapes; geoboard
Short description	The lesson is to be carried out after introducing the shapes by making prints of the faces of the 3D geometric solids. In this unit students: •recognise and name basic geometric shapes, •make instructed observations, use multiple senses, draw or write what they observe, •familiarize themselves with the geoboard, •create the shapes on the geoboard, •understand the simple rules of elementary games by recognising and naming geometric shapes.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: Mother tongue A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: recognising and naming basic geometric shapes
	A2: observing shapes in nature
	A5: elementary games, natural forms of movement
	A7: recognising and naming basic geometric shapes in English
Expected prior knowledge	Geometrics solids
Expected learning outcomes	MATHEMATICS • recognises and names basic geometric shapes, • creates the shapes on the geoboard. • makes instructed observations, uses multiple senses, draws or writes what he/she observes ENGLISH • recognises and names basic geometric shapes in English (circle, triangle, rectangle, square) • understands the simple rules of elementary games by recognising and naming geometric shapes, SPORT • practices natural forms of movement (different forms of walking, running, elementary throws, jumping, climbing, crawling, rolling, etc.)
Expected duration	90 minutes
Preparation	The following should be prepared: 7. shape models, 8. bingo cards, 9. pictures of shapes on geoboards, 10. geoboards, 11. rubber bands, 12. pictures of shapes

	13. “Make a circle” song https://www.youtube.com/watch?v=ALcL3MuU4xQ
Detailed description of activities	**1. INTRODUCTORY PART – ENGLISH LESSON** Start the English lesson with the song “Make a circle”. The teacher then introduces the shapes to the learners by showing them and naming them in English. (He/She uses movement: **It's** - taps thighs with hands; **a** - claps hands or snaps fingers; **circle/triangle/rectangle/square** - makes a circle/triangle/rectangle/square with fingers. Then they hold hands again in a circle and the teacher says: "Make a triangle/square/circle/rectangle." Together they try to make shapes. Each learner is given a shape (Appendix 1) and then follows the instructions of the teacher, who names the shapes in English - they work in a semicircle, standing up (e.g. Show me a circle. Show me a triangle.). After 4 activities, the pupils swap the shapes in the semicircle, followed by new, similar activities (If they know the instructions for movement in English, the instructions could be e.g. If you have a triangle, jump. If you have a square, make a squat, etc.) **2. MAIN PART** Work is done in groups of e.g. 4 learners. - If this is the learners' first contact with the geoboard, then before creating the shapes on the geoboard, the learners can create a figure of their choice on the geoboard and name it[1] . First, they form a figure with one rubber band and name it. Then give the pupils another rubber band to complete the original figure and name it. They then rotate the geoboard 180 degrees and see if the figure shows the same image as before or if the image has changed. - Each learner pulls out one shape from the bag (the shape used in the introductory game, Appendix 1) and displays it on the geoboard. When all the pupils have formed a shape, the teacher asks them to name the shape they have formed (they can also name it in English). Discuss with the learners whether it is possible to form a circle on the geoboard. The learners rotate the geoboard 180 degrees and say whether the image on the geoboard still shows the same shape. Then the pupils choose another shape to display on the geoboard and name it. - Creating shapes from memory - the teacher tells the pupils which shape to create on the geoboard. (Can also be done in English.) - Creating two-shape layouts on a geoboard based on pictures (Appendix 2) - each learner (or pair) is given a picture of two shapes and tries to recreate it on the geoboard. The teacher checks the solutions, and the learner names the two shapes he/she has created. They then exchange the cards. - Each learner is given a bingo card with 4 shapes (Appendix 3). The teacher draws the shapes out of the bag, names the shape and its colour, and if the learners have it on their bingo card, they cross it out. The first learner to cross out all the shapes and shout "Bingo" is the winner. (If the activity is done in English, they must also revise

[1] The activity can also take place the day before the cross-curricular lesson.

	the colours first.) **3. CONCLUSION** Physical activity takes place in a gym or outdoors 1. Elementary game: relay games to develop speed/strength and/or coordination with shapes (also suitable for the main and final part of the Sport lesson) The learners are divided into groups of 4. On the other side of the gym, a black and white picture made of shapes is placed in a ring. On the teacher's signal, the first learner in the line runs (jumps, crawls, etc.) to the other side of the gym to the ring with the shapes picture and a larger number of shapes. The pupil takes only one shape and puts it on the part of the picture (I suggest that there is velcro on the picture and the shapes so that they stick together). Once the learner has pasted the shape on the picture, he/she runs back to the first learner in the line and passes the baton to him/her, who then repeats the task. The learners carry out the task until they have filled the whole picture with the shapes. The first team to complete the picture with the coloured shapes wins. Repeat each relay game 2 times. (Example Appendix 4a) ● The task can also be made more difficult by asking the learners to count and write how many shapes are in the picture (Appendix 4b). In the next exercise, they use the given shapes to make a picture on their own, and at the end they tell what they have made and which shapes they have used. 2. Elementary game: make a picture using the shapes (also suitable for the introductory preparatory and main part of the Sport lesson) The teacher places/hides different coloured shapes on the floor and the lower equipment in the gym. The pupils are divided into groups of 4 and stand on mats placed at the edge of the gym. Each group is given a picture made of shapes which is placed on the mat. On the teacher's signal, the pupils run to the shape and bring it to the mat and see if it matches their picture. If it matches the shape in the picture, they put it on the mat and if it doesn't, they carry it back to the place where they took it. The first team to complete the picture with the coloured shapes wins. Repeat the game several times.
Extension activities	Additional activities can include working with a geoboard app.
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanić, Tadeja Volmut, Mojca Žefran (University of Primorska, Faculty of Education)

Title	**Weather**
Key words	Weather conditions, columnar and linear displays, predicting probability of an event
Short description	This activity requires students to monitor and record weather conditions and learn how to present their data and predict the weather. In this unit students: • record weather conditions and present their data in columnar and linear displays, • predict the probability of an event, • make a simple weather calendar, • learn about appropriate sports clothing and footwear, • learn how to talk about the weather in English.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: Mother tongue A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: columnar and linear displays, reading the displays, predicting the probability of an event
	A2: monitoring and recording weather conditions, making a simple weather calendar, comparing weather at different times of the year
	A5: appropriate sports clothing and footwear
	A7: English terms for weather conditions, weather symbols/pictograms, talking about the weather in English
Expected prior knowledge	Predict the probability, comparison.
Expected learning outcomes	**MATHEMATICS:** • presents data in columnar or linear displays, • reads the demonstrations, • is able to predict the probability of an event using the terms: possible/impossible or certain, **NATURAL SCIENCE:** • is able to monitor and record weather conditions, • knows how to make a simple weather calendar, • is able to compare weather conditions at different times of the year, **SPORT:** • learns about appropriate sports clothing and footwear, **ENGLISH** • learns about the English terms for weather conditions, • knows how to talk about the weather in English using the question “What’s the weather like?” and answer “It’s sunny/rainy/cloudy…”
Expected duration	90 minutes
Preparation	The following should be prepared (in the appendices): jigsaw puzzles, weather pictures, pictures of symbols for weather conditions, different weather forecasts, Memory game cards, weather recording worksheet
Detailed	**1. WARM-UP**

description of activities	Divide the pupils into groups of 4. Give each group a jigsaw puzzle (cut-up pictures showing different weather conditions - Appendix 1). They glue the individual pieces onto a piece of paper to make a complete picture. The groups put the completed pictures on the board using magnets. We discuss what the pictures show and what the weather is like. **2. MAIN PART** We introduce the symbols for weather conditions. Then we add the symbols to the pictures showing weather conditions we put together earlier. This is followed by a discussion with the learners, through which they learn about the characteristics of each weather condition. We also show the symbols. For example: - What is typical of each weather condition? Can the same weather condition occur at night? - What is the most common activity then? What sports can we do? - How are we dressed? (We also point out the differences in seasons - e.g. How are we dressed in summer when it is sunny and in winter when it is sunny? What time of year do you think the the photo of the girl with the umbrella is taken? How would she be dressed if it was raining in late autumn?) - How does such weather affect the lives of other living beings (e.g. Which animals are more likely to be seen when it rains? Is rainy weather important for plants? Why? ...) - For some weather conditions, we emphasize the potential hazards (e.g. thunderstorms-lightning bolts, hail, wind, etc.). We also give them the weather forecast for the next day (we show examples of weather forecasts in different media - TV, radio, the internet, newspaper). Then the pupils are given the task of trying to be meteorologists. They either work in groups or we can do the activity by choosing one learner to present to his/her classmates what the weather in Slovenia will be like the next day. A simplified example of a weather forecast:  After analysing the weather picture, we have a discussion related to the probability based on the weather forecast. Examples of questions: - Is it possible that it will snow tomorrow? Why do you think so? - Depending on the weather forecast, could there be a rainbow somewhere? - Is it necessary to have an umbrella with me according to the weather forecast? - Would it be possible to swim in the outdoor pool tomorrow? etc.

3. CONCLUSION

For revision, we can play either the "Hot seat" game or the "Memory" game.

Hot seat game: we choose one learner to sit in the chair in front of the board (with his/her back to the board, so he/she cannot see it). We project the individual weather symbols on the board. The other learners have to describe to the learner sitting on the hot chair what they do in this weather, how they are dressed... They have to be careful not to say the word indicating the weather condition. When the learner guesses the weather condition described by his/her classmates, another learner takes his/her place.

Memory game: we divide the pupils into pairs or small groups. Each pair or group is given the Memory game cards, with one card representing a symbol and the other a photo of the weather condition (Appendix 2).

When the game is finished, we tell the learners to observe the weather and record it in a weather calendar for the next 5 days – in the morning we observe and record the weather at school (always at the same time) and in the afternoon at home (Appendix 3). This will allow us to link the content to mathematics in the following lessons.

Analysis after five days of weather observations:

- What was the weather like on Monday/Tuesday/Wednesday, ... **afternoon**? (Why doesn't everyone have the same weather?)
- Which weather condition occurred most often **in the morning**?
- How many times in one week did you mark e.g. cloudy weather?
- Did it rain on any days of the week? Which day? In the morning or afternoon?

You can also work with the learners to complete the table in Appendix 4 - How many times did each weather condition occur? Discuss which weather occurred most often, which least often, which did not occur at all, why not, ... Ask the learners if they can tell from this table what the weather was like on which day or during which part of the day.

THE ENGLISH LESSON

1. The teacher asks the pupils, *What's the weather like? The* teacher asks the pupils to show the sun with their hands (draw a big circle in the air*)* and say: It's sunny. Then the teacher calls one learner to the front of the board and asks him/her to do pantomime and show that it is raining and asks the pupils: *What's the weather like?* The pupils say what their classmate is showing and the teacher says the answer in English. They do this for the following weather conditions: sunny, snowy, cloudy, windy, rainy, (foggy). Each time a different learner comes to the front of the board.

2. Then the teacher sings The weather song (below; tune: Oh my Darling Clementine):

(https://www.youtube.com/watch?v=sq9eCcLkMmI)

What's the weather?
What's the weather?
What's the weather like today?
Tell us SOPHIE (/Anja, Peter...)
What's the weather?
What's the weather like today?

Is it sunny?

Is it cloudy?
Is it rainy out today?
Is it snowy?
Is it windy?
What's the weather like today?

Sing the song together and include movement.

3. The teacher shows the learners the symbols of the weather conditions (sunny, snowy, cloudy, windy, rainy, foggy) and they name them together.

BINGO game: symbols are used to play BINGO.

4. The weather wheel: learners work in groups (or pairs) to make a weather wheel (glue symbols for weather conditions on paper plates, cut out a pointer from cardboard and attach it to the centre of the circle). Example of making a weather wheel: https://conservamome.com/weather-indicator-paper-plate-craft/

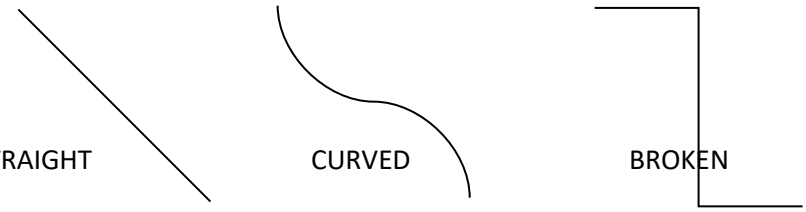
Then one of the learners sets the weather on the wheel and asks a classmate *What's the weather like today*? The classmate answers *It's* ____. They repeat the activity several times, switching roles in between.

5. Additional activity: if the learners already know the terms for weather conditions or days of the week, they can also record the weather in one week (make a weather calendar in English; see the example of a weather calendar in the general preparation for this cross-curricular unit).

As an introduction to this, we can do the following listening activity with the learners:
Read the following text to the pupils and ask them to cross out the symbols in the corresponding squares:
On Monday it will be warm and sunny. A perfect day to go swimming.
On Tuesday it will be windy. Make sure to put on some warm clothes.
On Wednesday it will be cloudy but it will still be warm.
On Thursday it will be rainy so don't forget your umbrella.
On Friday it will be foggy. Be careful on the road.

MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY
●○○○○	○●○○○	○○●○○	○○○●○	○○○○●

Extension activities	
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanić, Tadeja Volmut, Mojca Žefran (University of Primorska, Faculty of Education)

Title	**Straight and curved lines**
Keywords	Straight, curved, broken lines, dots, letters
Short description	As part of the activity, students will get to know the concepts and know how to recognize straight, curved and broken lines, as well as the concept of a point in mathematics and connect them to the different capital letters they have learned and the small letters they are currently learning from their mother tongue.
ICT tools included	3d printer
Areas (select)	**A1: Mathematics** **A2:** Natural science A3: **Art culture** A4: Musical culture A5: Physical culture A6: **Mother tongue** A7: Foreign language Other: **A8:** Informatics A9: Civic education
Topics (for each selected area)	A1: Straight, curved and broken lines
	A3: drawing, recognizing objects in space
	A6: capital and small print letters
Expected prior knowledge of students	Recognize of the alphabet and numbers, recognition of basic geometric shapes and bodies
Expected outcomes	Recognize of straight, curved and broken lines, understanding of intersections Determining the course of time and time intervals in everyday situations
Expected duration of the activity	90 minutes
Preparation of activities	Prepare a worksheet for identifying straight, curved and broken lines Prepare models of capital letters made with a 3D printer
Detailed description of all teaching activities	In the introductory part of the activity, the teacher shows the students a drawing of a landscape on which she explains the relationships of objects, the appearance and shape of objects. After that, he tells them that all the objects in the picture are drawn using curved, straight and broken lines. Then he draws one curved, straight and broken line for them on the board and describes them.  STRAIGHT CURVED BROKEN The teacher asks the students if the shapes of the lines remind them of the individual printed letters they have been studying. Through the interaction, the teacher explains that printed letters (they

	have already learned large letters, and they are currently learning small printed letters) are written using straight, curved and broken lines. He also states that all objects around them are made of straight, curved and broken lines and surfaces (which he will learn later). He cites geometric figures as an example; a circle bounded by a curved line or a triangle bounded by straight lines. After that, he asks the students who wants to go in front of the board and write a letter they know that is made of a curved line. ***Example of letters C, J*** After that, he takes another student for an example of a printed letter made of a straight line ***Example of letters I, T*** Then he takes the student for an example of a letter made from a broken line ***Example letter Z, V*** Hand out worksheets to the students that contain examples of different lines, and the students should name them. In the second part, the teacher explains to the students that the lines intersect and that the intersection of the lines is marked with a point. A POINT (.) is marked with a capital letter (A, B, C, D, ...) He asks them where else they use the point. They are reminded that a full stop is used at the end of a declarative sentence. They also mention that in addition to the full stop, a question mark and an exclamation mark are used in the language, so he asks them what kind of lines those two punctuation marks are made of. ? (QUESTION POINT) – curved line ! (EXCLAMATION MARK) – straight line After that, the teacher explains that the shortest connection between two points is called a length, where, for example, points A and B are the end points of that length. This introduces them to the next lesson. For the homework, the students have to create a short poem about lines, which they will write in a notebook in capital letters, and identify in the poem which lines are used to write the letters/words in the poem.
Possibilities to expand activities	Creating straight, curved and broken lines using a computer application
Additional notes	
Authors	Antea Čilić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Storytelling – mathematical edition**
Key words	reading comprehension; description of a figure; solid and plane shapes; months; seasons
Short description	Native language, mathematics and natural science contents are connected through game, reading, and telling mathematical stories. By reading mathematical stories, students strengthen their reading literacy, and by analysing the text and describing mathematical objects, students notice the properties of an individual object, distinguish between objects, and recognize the characteristics that uniquely describe each object. By creating stories and describing mathematical objects, students develop creativity, describe, and write down their observations, and practice writing techniques and rules. The activity can, using the same concept, be adapted to different mathematical content, and additionally supplemented with activities within foreign language class.
IT tools	3d print
Fields (select)	A1: **Mathematics** A2: **Natural science** A3: Art A4: Music A5: PE A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: solid and plane shapes
	A2: months of the year and seasons
	A6: reading comprehension; storytelling; description of a plane shape
Expected prior knowledge	solids, plane shapes, length, edge, side; months; seasons; reading comprehension; rules for describing a plane shape
Expected learning outcomes	• Adds and subtracts in a set of natural numbers up to 100 • Multiplies and divides within the multiplication table • It applies four computational operations and relations between numbers. • It connects known geometric objects. • Uses data from the immediate environment. • Determines whether an event is possible or impossible. • The student concludes about the changes in nature that occur during the seasons. • The student explains the organization of time and displays the timeline of events. • The student reads short texts thematically appropriate to the student's experience, language development and interests • The student expresses his observations, thoughts and feelings after listening to/reading a literary text and connects them with his own experience.

	• The student creatively expresses himself according to his own interest, encouraged by various experiences and experiences of the literary text. • The student talks and speaks in accordance with the theme of everyday life and respects the rules of polite conduct. • The student describes on the basis of observation, carefully and politely listens to the interlocutor without interrupting him in speaking. • The student writes in the school handwritten letters, words and short sentences in accordance with language development.
Expected duration	90+45 minutes
Preparation	1. Prepare and print a mathematical story 2. Story Game Platonic Solids a. Platonic solids can be prepared using origami techniques (https://mathigon.org/origami/cube) or 3d printer (https://www.tinkercad.com/things/gYYsLM0xlXJ) b. Stickers (https://inamath.uniri.hr/wp-content/uploads/2022/03/story-solids.pdf) • 4 seasons • 6: less, equal to, greater than, times 2, minus 2, plus 2 • 8 geometric shapes: triangle, circle, square, rectangle, curved line, straight line, broken line, length • 12 months • 20 numbers  3. Prepare and print a worksheet on which students will write their story.
Detailed description of activities	PART 1: Math story (90 minutes) In the introductory part, as a motivation to continue the activity, students are asked if they like to read stories? Do they like to listen to stories? And tell stories? What kind of stories do they like to tell? Have they ever heard a math story? 1. Reading comprehension (30 minutes) In this activity, students read the mathematical story The Mathematician Owl (https://inamath.uniri.hr/wp-content/uploads/2022/11/Mathematical-story-Owl-the-Mathematician.docx) and answer the questions after reading the text. Students are given a piece of paper with a story and questions, and they are asked to answer the questions in full sentences, paying attention to the rules of writing, and to indicate a mathematical calculation if it is needed to answer the question. After the students have read the text and written down the answers, they all read the questions and answers together and discuss whether they are correct and analyse the parts of the text where the answers are hidden. Since solids are mentioned in the text, it is desirable to have examples of solid

shapes if it is necessary to analyse them.

2. Game with solids for telling stories (30 minutes)

Each pair of students is given a set of 5 Platonic solids.

In the introductory part students are introduced to the props they will use. They are asked if they recognize any shapes, if they know what dice are, what games they played with dice, ... Instead of dice, it is explained to them that today we will use solids, special solids that we call Plato's solids in honour of a great Greek scientist and the founder of the Academy.
The rules of the game are explained to the students below.

The game can be played in several ways:

a. The players are assigned a story topic in advance (e.g., s field trip, a sport, a game, school, in the nature...) or one of the players assigns a story topic. After throwing 5 solids, each player tells a story on a given topic using the pictures they get (we notice that 4 out of 5 Plato's solids have the upper side, except for the tetrahedron, so the players should be instructed to look at the picture on the lower side of the tetrahedron, which gives an additional dynamics of the game because the players do not see the picture in advance or they need to deduce what the fourth concept is from the visible 3 concepts).
b. The players start the story with the words "Once upon a time" and after throwing 5 solids, tell the rest of the story using the pictures shown on the solids.
c. A pair of players divides four solids among themselves and tells the story, continuing each other, based on the images they get.

Students play the game in pairs. Teacher monitors, listens to their stories, helps them, corrects if necessary...

3. Write your story! (30 minutes)
    a) Together with students, the teacher creates a story and writes it on the board. First, the teacher writes down the title of the story (eg A Field Trip), it can be the same title that the students used to tell the story in the previous activity. Then one of the students throws the solid shapes. The teacher writes/copies the concepts on the board based on which they write the story. Then they come up with 5 sentences of the story together.
    b) Students working in pairs are given the following task. They use a worksheet prepared by the teacher (https://inamath.uniri.hr/wp-content/uploads/2022/11/worksheet-Write-your-story.docx). A pair of students throws the solid shapes, and each student writes down/copies concepts on a piece of paper and writes down a story that is related to the pictures in the game. The story must have at least 5 sentences. Each student writes three questions related to the story below the story.
4. For homework, students finish their stories and questions. At the beginning of the next lesson, students (in pairs) change the stories, read the story of another student and answer the questions. In the end, students check whether the answers are correct.

PART 2 (in the next lesson): Description of a geometric object (45 minutes)
The teacher prepares a set containing plane shapes, solids, and places them

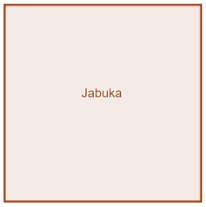
	where they are visible to all students. Each student, in secret, chooses one element from that set to write a composition about. Similar to describing a literary character, students are given guidelines for description. I. Is your object a plane or solid shape? II. If it is a plane shape, how many vertices and sides does it have? What are its sides like? III. If it is a solid shape, how many vertices, edges and sides are there? What are its edges like? What are its sides? IV. Does your object have any other unique features that you have not written? It is important to emphasize to students that their description should be such that someone can recognize which object it is. Students have 15 minutes to write a description of the geometric shape they have chosen. While writing, students are free to walk to take a closer look at the exhibited object. After that, students take turns reading their descriptions and other students try to guess which object they are describing. This part of the activity can also be carried out by dividing the students into several groups and placing a set of geometric objects on the desk for each group, if we have prepared more than one set of plane shapes and solids. It is very important to tell students if they have not unambiguously described a geometric object from the set of offered objects, and to guide them to describe the differences between the objects (e.g. if a student writes "my object is an angular solid, and in the set there is a cuboid and a cube , it is emphasized to them that both the cuboid and the cube correspond to their description, and the student is instructed to use an additional sentence to emphasize the properties characteristic only of the object they imagined). For homework students can correct their descriptions if they want and write a composition on one of two topics. • Describe a cube, describe a cuboid and describe the difference between a cube and a cuboid • Describe a square, describe a rectangle and describe the difference between a square and a rectangle
Extension activities	1. This scenario has been prepared as an activity on the topic of plane and solid shapes. However, the activity can be designed on any topic. The activity can be carried out regularly in all classes, with topics and questions that correspond to the content that is currently being taught (it can be used either when learning new content or when revising). 2. In the activities of this scenario, the support of the English language teacher would be very useful. The activities could be modified in such a way that the story is told in English. This way the students, on top of all the above, would also learn English terms and mathematical concepts that are part of the story.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Describe me and place me!**
Key words	Solid and plane shape, set, syllable, vowels, consonants
Short description	The activity is based on content of sets, but it can also be carried out with students who are not familiar with the concept of set or as motivation for introducing the concept of set. Tasks aim at operational acquisition of the definition of terms, noticing the differences between terms, noticing the common characteristics of the terms. In the activity described below examples of tasks were given on the topic of solid and plane shapes, as well as syllables, vowels, and consonants. Similar activities can be carried out with all ages, in all subjects and on different topics.
IT tools	3d print
Fields (select)	A1: **Mathematics** A2: Science A3: Art A4: Music A5: PE A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: sets, solid and plane shapes
	A6: syllable, vowels, consonants
Expected prior knowledge	solid shapes, plane shapes, length, edge, side; syllables, vowel, consonant; set (it is desirable that students know the concept of a set, but it is not necessary; this activity can be used to introduce the concept of a set in the educational systems where the concept of a set is not part of the mandatory elementary school curriculum)
Expected learning outcomes	• It connects known geometric objects. • The student talks and speaks in accordance with the theme of everyday life and respects the rules of polite conduct. • The student writes in the school handwritten letters, words and short sentences in accordance with language development.
Expected duration	90 + 90 minutes
Preparation	5. Preparing shapes for the main activity can be done in different ways, we list some of them: • print the materials for the activity on a 3d printer (https://inamath.uniri.hr/wp-content/uploads/2022/05/3d-Print.rar), • prepare wooden or plastic models, • the teacher prepares a model in Tinkercad with the students, through a simple activity in which children recognize shapes and change their colour and size, • students bring solid and plane shapes from home according to the teacher's instructions, • in art classes students can create shapes following the teacher's

	instructions, for example, they can do origami, stick from a network of solids, make symmetrical plane shapes, ... It is necessary to make sure that all elements are different from each other (if, for example, two solids are the same, we can make them different by sticking a sticker or making some other mark on one of them) to avoid unnecessary confusion (the elements of one set are always mutually different). This activity is designed and prepared for a universal set that contains: a blue triangle, a blue square, a red plane shape that has 4 sides of equal length, a red plane shape that has 4 sides that are not all of equal length, a red plane shape that has 6 sides, a blue shape that has 6 sides, red shape that has 7 sides, blue shape that has 8 sides, blue cone, red cylinder, blue cuboid, red pyramid that has 4 sides, blue pyramid with 5 vertices, red pyramid that has 10 edges, red sphere, blue round solid with a hole (e.g. torus), a blue angular solid that has exactly two triangles as sides, a red angular solid that has 14 vertices, a blue angular solid that has 15 edges, a red angular solid that has 14 sides, a blue angular solid that has 12 vertices, a red angular solid with 10 sides. 6. Place me and describe me playing cards (https://inamath.uniri.hr/wp-content/uploads/2022/11/cards-for-sets.xlsx) In one set, which contains elements of the universal set, there are 10 pairs of cards: one card contains a description of the elements of the set, and the other a list of the elements of the set. 7. Memory game cards (https://inamath.uniri.hr/wp-content/uploads/2022/11/game-cards.docx) For each geometric object that belongs to the universal set, a pair of cards is prepared: on one card there is a picture of the object, and on the other a description of the object. 8. Pictionary game cards (https://inamath.uniri.hr/wp-content/uploads/2022/11/game-cards.docx) Some mathematical terms or some geometric object from the universal set are written on the Pictionary game cards. The student should draw a term from the card. 9. Math alias playing cards (https://inamath.uniri.hr/wp-content/uploads/2022/11/game-cards.docx) The student should explain the term from the card without using the words or the root of the words written on the card. There are plane shapes from the universal set on the cards. 10. Game instructions There are short instructions for the games provided in the scenario: Memory, Math alias, Pictionary, Guess what I imagined, Place me and describe me
Detailed description of activities	As preparation for the activity in class, the scenario *I am telling you a mathematical story* can be carried out. Let's play with sets (90 minutes) 1) Getting to know the props (5 minutes) We put the elements of a set inside the rope, then distribute the ropes (tied ends) to the students. Each group of students is given a set of some geometric shapes. In the following tasks, we will extract its subsets from this set according to the given criteria. However, before we start the tasks, we give the students some time to play with the new props. 2) Select the elements of the set (15 minutes) Tasks like the following (the task itself depends on the objects prepared by the teacher for the lesson, in this scenario the tasks are designed with the

	assumption that the specified universal set is used). a) Put all blue geometric figures inside the set. Does your set contain a triangle? b) Put all the circular blue solids inside the set. How many elements does your set have? c) Put all the red polyhedron inside the set. Is any pyramid outside the set? d) Put all figures that have four or six sides inside the set. Are all figures the same colour? e) Put all solids that have less than 10 edges inside the set. Are all elements of a pyramid set? f) Put all geometric shapes that have less than 9 vertices inside the set. How many solids are there in the set? g) Put all solids inside the set that have at least one side that is a rectangle. How many solids are there that are not pyramids? The teacher, considering the reaction of the students, chooses whether to solve all the examples or only a part of them. 4) Intersect two sets (15 minutes) Students are given two different colour ropes (e.g., red, and blue) which they place next to each other. We assign the following tasks (the tasks depend on the prepared materials, i.e., the universal set): a) Put all the red plane shapes inside the red rope put, and all the blue solid shapes inside the blue. b) Put all the pyramids inside the red rope, put all the blue polyhedrons inside the blue rope. c) Put all the shapes that have less than 7 vertices inside the red rope, and all the polyhedrons that have at least one side that is a rectangle inside the blue rope. d) Put all the solids with at least one side of a triangle Inside the red rope, and inside the blue all the red polyhedrons. e) Put all the polyhedrons that have more than 5 sides inside the red rope, inside the blue rope all the plane shapes that have more than 7 sides or all the solids that have more than 12 edges. Students, with the help of the teacher, should conclude that in certain tasks they should overlap two ropes, that is, make a cross section of sets. The teacher, considering the reaction of the students, chooses whether to solve all the examples or only a part of them. 5) Determine the common property of all the elements written on the card (20 minutes) Note: This task is difficult for students, but it can be done with students in additional math classes. If activities like this one are regularly carried out in class, students will gradually learn to solve such tasks without difficulty. Students are given 3 cards with a list of subset elements. The students are expected to find all the elements and put them inside the rope and to complete the sentence: "The set contains all...". In our case, we will distribute cards to students (tasks go from simpler to more complex) on which the following is written: a) Assemble a set containing cones, cuboids, pyramids with 5 vertices, solids with a hole, solids with exactly two triangles as sides, solids with 15 edges, solids with 12 vertices. Criterion: solids of the same colour b) Assemble a set containing cuboids, solids with at least one side being a triangle, solids with more than 13 edges. Criterion: polyhedrons c) Assemble a set containing triangles, plane shapes with 4 sides

	of equal length, plane shapes with more than 6 sides. Criterion: plane shapes that have all sides of the same length. This task is extremely difficult for the students and the teacher actively monitors and tries to help the students in determining the criteria. For example, if the student answers: "The object inside the rope is a polyhedron that is red or blue in colour", and there is a pyramid left outside the set, the teacher should add the pyramid to the set and in this way indicate to the student that the description is not precise enough. 6) Sets with words (20 minutes) The activity continues in the mother tongue class. The teacher writes on the board a set containing the names of all the students in the class (in the form of a Venn diagram) and writes down the criteria by which he selected the elements of the set (e.g., the set of all words that are students' names). It should be emphasized here that not a single student should be left out, because we want to write down the elements of a set that contains the names of all students, but also that one name should be written only once in the set, even though there may be several students with the same name. Note: the elements of the set can also be selected according to some other criteria, for example, the words we use to name the colours you see in the classroom, the words that are the names of your teachers, the words we use to name the items in the pencil case, ... Students are given the following tasks. Write the elements of the subset containing: a) words that have exactly three syllables, b) words that have two opening words, c) words that have at least 3 and no more than 6 letters. 7) Assignment: Students are given a similar assignment for homework. In your notebook describe one set of words and in the Venn diagram, write the elements of that set. If the same activity is to be continued in the next lesson, the students are given the task of preparing pieces of paper on which to write the elements of that set. With pieces of paper, similar tasks can be continued in the next lesson, revising concepts from the mother tongue. Additional activity: Play and define (45 - 90 minutes) Note: this part can be part of an integrated or project day and can include a storytelling game with storytelling solids. Classroom preparation: In the classroom, prepare 6 stations that are named (e.g., after fruit trees as in the picture).



Each student chooses whether to play Memory or Pictionary. Furthermore, each student chooses one of the 3 following games: Guess What I Imagined, Math Alias, and Place Me and Describe Me.
Students write down their choices in their notebook (or on a piece of paper).

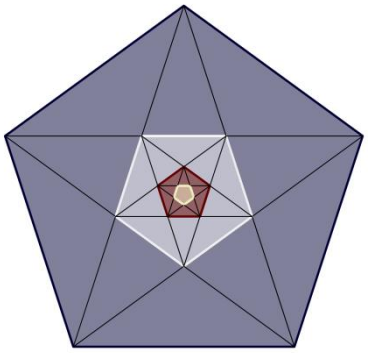
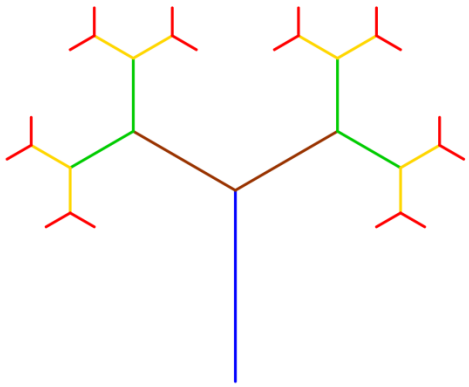
The following is written on the board:
• The Pictionary game is played at the stations whose names are in the following set: {Plum, Banana, Lemon} (the teacher can write the elements of the set in a Venn diagram on the board).
• The Memory game is played on stations whose names are in the following set: {Apple, Pear, Strawberry}
• The game Guess what I imagined is played at the stations whose names are in the following set: {Lemon, Strawberry}.
• Math alias game is played on stations whose names are in the following set: {Banana, Pear}.
• The game Place me and describe me is played at stations whose names are in the following set: {Plum, Apple}.

All the students get up from their seats and have to find the station that corresponds to their choice of games.
In an analogous way, the activity can be carried out outside, on the school playground, in the school hall, ... and in a similar way, activities and division of students into groups can be assigned in other subjects as well (e.g., in the school hall during PE class, in art class, etc.).
It is desirable that the teacher corrects students if they describe geometric shapes imprecisely or if they express themselves incorrectly, which is difficult if students play in groups. However, all games (except the Memory game) can be played by the whole class together, i.e., one student "against" all the others.
Game descriptions
Notes:
• each game is played by two students;
• in all games, the elements of the universal set with which the previous activities were carried out are used as help;
• the rules of the game can be adjusted in such a way that one game is played by several players, that points are counted, etc.:

1. Guess what I imagined: One student imagines a concept from a set of objects on the table. The second student asks questions that are answered with YES and NO until he guesses which object it is. Students

	play in turns. 2. Math alias: One student takes a card and tries to explain the term written on the card, but in such a way that they must not mention the term itself or similar words. Students play in turns. 3. Memory: Students arrange the cards upside down and take turns opening two cards. 4. Pictionary: One student takes a card and tries to draw the term written on the card, the other student should either say what is written on the card or show an object from the universal set. Students play in turns. 5. Describe me and place me: Students draw a card with a list of geometric shapes they must find and together try to come up with a criterion by which the set is composed. We can turn this activity into a game as follows: Inside a cardboard box (hidden so that the other student cannot see), one student prepares the elements of the set according to the card. The student has to explain to his partner what is inside his box without naming the objects.
Extension activities	3. This scenario has been prepared as an activity on the topic of solids and figures. That is, the universal set contains plane and solid shapes. However, elements of the universal set can be: a. numbers, e.g., numbers up to 100, and properties that define belonging to a set, e.g., even numbers, multiples of 3, divisors of 30, greater than 50, numbers that are given as results of computational operations, images that have a certain number of the same type of buildings, ... b. letters or words, and criteria related to content in the language (native or foreign): vowels, consonants, nouns, verbs, conjunctions, abbreviations, ... c. articles of clothing, and criteria related to nature, for example, the seasons; or jobs that are done in certain seasons or animals that sleep or do not sleep in winter, d. … 4. A similar activity can be carried out with students from the 1st grade, with a reduced number and complexity of tasks. The activity can be carried out regularly in all classes and in all subjects, with tasks that are assigned in a way that corresponds to the content that is currently being taught (it can be used either when teaching new content or when revising what has been learned). 5. In additional math classes, the activity can be used for various tasks in which students actually work with functions (although they do not need to know that they are working with functions), injections, surjections, bijections, ... E.g., students are asked how they can compare the number of elements in two sets, without counting the elements of the set. 6. For older age groups, the concept of union can be introduced in a similar way, and by counting the elements of a set, you can arrive at the sum principle, inclusion, and exclusion formulas (for 2 or 3 sets), De Morgan's formulas... 7. In the activities of this scenario, the English teacher's support would be particularly useful, and in this way the students could also adopt English terms for plane and solid shapes.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Look at me… I repeat myself!**
Key words	fractal, multiple, deciduous and coniferous trees
Short description	As part of this activity, students are introduced to the concept of fractals and, following the instructions step by step, draw simple fractals and build a fractal tree. Through tasks related to the fractal tree, students practice counting to 100 and learn or repeat the concept of multiples. Students compare the appearance of a fractal tree with the appearance of deciduous and coniferous trees and use the observed regularities to draw trees in the art class. Although fractals are complex, there are many simple fractals that students can draw independently using simple rules. Activities that include fractals are an excellent way to develop algorithmic thinking and focused execution of procedures according to exactly given instructions. It is possible to continue the activity in IT and art classes.
IT tools	GeoGebra; 3d print
Fields (select)	A1: **Mathematics** A2: **Natural science** A3: **Art** A4: Music A5: PE A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: multiplication, multiples, length units (cm) A2: deciduous and coniferous trees A3: drawing with a pencil
Expected prior knowledge	multiplication and division up to 100; multiples (the activity can be used to introduce the concept of multiples); deciduous and coniferous trees
Expected learning outcomes	• Uses natural numbers up to 100 to describe and display quantity and order. • Multiplies and divides within the multiplication table. • Estimates, measures and draws the length of the given length. • The student concludes about the organization of nature. • The student concludes about the changes in nature that occur during the seasons. • The student demonstrates knowledge of the peculiarities of various visual materials and procedures in artistic expression. • The student uses some of the proposed art materials and techniques.
Expected duration	90 minutes + 45 minutes
Preparation	11. Worksheet (https://inamath.uniri.hr/wp-content/uploads/2022/11/worksheet-tree.docx) 12. Fractal trees - preparation for 3d printing (https://www.tinkercad.com/things/94cmF3z69Qa) The activity is carried out as group work. In total, per group, at least 61 sticks, 15 branches and 26 flat paper clips are needed

	13. Paper on which a regular pentagon is drawn (https://inamath.uniri.hr/wp-content/uploads/2022/11/peterokut.docx) 14. A GeoGebra file to follow the creation of a fractal that starts with a regular pentagon (https://inamath.uniri.hr/wp-content/uploads/2022/11/peterokut2.ggb) 15. GeoGebra files to follow the creation of the fractal tree (https://inamath.uniri.hr/wp-content/uploads/2022/11/stablo-za-slaganje.ggb)
Detailed description of activities	1. Look at me… I repeat myself! (30 minutes) What are fractals? Fractals are self-similar objects. We create self-similar shapes by constantly repeating the same process of building the object forever. Students, following the step-by-step construction in GeoGeoGebra, draw a simple fractal. The students are given papers on which a pentagon is drawn, and they have to draw the diagonals. In order for the students to do better and draw diagonals more easily, they need to be told to point out the vertices of the pentagons in each iteration and that these are the connecting points.  Below, students are shown images of some other fractals. 2. Are there fractals in nature? (5 minutes) Students are shown pictures of some fractals in nature, with a note that there are no mathematical fractals in nature, but that there are many shapes that are similar enough to mathematical fractals. Special emphasis is given to trees, deciduous and coniferous trees are distinguished because trees are built using different rules. This part of the activity can be planned and carried out in advance. Students can, either independently or together with the teacher, observe trees in their surroundings and see how they are "built". 3. Assemble your tree! (40 minutes) Students are given "toys" and given some time to play with them. Next, the students, in groups of 4-5, assemble the tree. The longest branch of tree is composed of 5 sticks, and each subsequent iteration has one stick less. 

	Students are shown a construction made in GeoGebra, which is shown step by step. Students can arrange the tree in two ways. • Students arrange the tree from the largest branch to the smallest. In this way, students can more easily observe and describe the process of creating fractals. • Each student in the group arranges his "small tree" (e.g., 4 trees with 3 iterations each) and then together they combine that tree into a larger tree. After two groups have finished their tree, they can build another, bigger tree (whose longest branch will be 6 sticks long). If you start with several smaller trees that join into larger ones, then all students participate in the construction because they do not disturb each other. However, it is more difficult to describe the way fractals are created in this way. 4. Explore your tree! (15 minutes) After assembling the tree, students are given worksheets to fill in. The teacher and students notice that multiples of the numbers 3, 1, 2, 5 appear in the tables. Note: this activity can be used to introduce the concept of multiples, but also for revision. Students solve the remaining tasks on the worksheet. 5. Draw your tree (45 minutes) In this part, students draw a tree using a pencil (or another technique), trying to follow the "algorithm" of creating a fractal tree. Students solve the remaining tasks on the worksheet.
Extension activities	Drawing fractals on the computer (e.g., GeoGebra, Logo, Scratch, Tinkercad) 1. Fractal tree 2. Golden tree 3. Pythagorean tree 4. Golden spiral 5. Fibonacci spiral 6. Sierpinski carpet 7. Sierpinski triangle Students can draw all the above in art classes using different techniques. Fractals can be used very simply and purposefully in teaching various math content, such as plane shapes, area of plane shapes, perimeter of plane shapes, units of measurement, etc.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Math card tricks: Pirate and treasure**
Key words	math card trick, counting to 100, even and odd numbers
Short description	The activity is based on math tricks with cards which develops a positive attitude towards math in students. This activity is an excellent way to strengthen pre-math and math skills in a fun and creative way that increases students' interests and motivation. Math background of the Pirate and Treasure trick involves counting to 100 and recognizing even and odd numbers. It is important to come up with an interesting story that accompanies the trick, and by creating a story, students develop creativity and the ability to tell a story. It is possible to connect the activity with contents from natural history. The activity develops students' motor skills, the ability to follow and reproduce the procedure and the focused implementation of the procedure according to exactly given instructions, in which the final success depends on the successful implementation of each individual step. It is possible to expand the activity by creatively designing performances within mother tongue, foreign language, art and music classes.
IT tools	
Fields (select)	A1: **Math** A2: Natural science A3: **Art** A4: Music A5: **PE** A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: pre-math and math skills (repeating the procedure in an exact order); counting to 100 (subtraction and division), even and odd numbers
	A3: drawing pirates and treasure
	A5: motor skills and fine motor skills in hands
	A6: creating a story; communication with the audience; giving clear instructions
Expected prior knowledge	counting to 100; even and odd numbers
Expected learning outcomes	• Uses natural numbers up to 100 to describe and display quantity and order. • Adds and subtracts in a set of natural numbers up to 100. • Multiplies and divides within the multiplication table. • The student compares his artistic or visual work and the works of other students and describes his own experience of creation. • He collaborates in elementary games and accepts the rules of the games. • He actively participates in elementary games that develop self-esteem, self-confidence and perseverance.

	• The student reads/listens to short texts thematically appropriate to the student's experience, language development and interests • The student talks and speaks in accordance with the theme of everyday life and respects the rules of polite conduct. • The student finds the data in the read text according to instructions or questions. • The student creatively expresses himself according to his own interest motivated by different experiences and experiences of literary text
Expected duration	90+45 minutes
Preparation	3. Prepare decks of cards (one deck per two students) 4. The teacher practices the trick (according to the instructions: https://inamath.uniri.hr/wp-content/uploads/2022/11/Instructions-for-the-trick-pirate-and-treasure.docx) 5. The teacher learns the math behind the trick (according to the explanation given in the scenario) 6. Worksheets for homework and additional activity (the story of the Pirate and the treasure that accompanies the trick): https://inamath.uniri.hr/wp-content/uploads/2022/11/The-story-of-pirate-and-treasure.docx 7. Scheme for the story that follows the trick: https://inamath.uniri.hr/wp-content/uploads/2022/11/branchingA4.pdf
Detailed description of activities	PART 1: Presentation of the trick by the teacher (25 minutes) The teacher announces that a math card trick will be performed. To perform the trick, it is not necessary to have dexterous hands, but to let the math do the magic part. At the beginning of the presentation, the teacher explains what a math card trick means: in math card tricks, the trick relies exclusively on math, without the necessary skills of the trick performer, without "fake" and marked cards and hidden information. In the introductory part, it is emphasized that the most important part of today's activity is the following. • Students listen carefully to the teacher and follow the instructions. • Students carefully, slowly, and concentratedly carry out everything the teacher tells them. • Students do not play with the cards but stop after each individual step and wait for further instructions. The teacher performs the trick (according to the instructions) without explaining it in detail. It is important that the teacher and students are positioned so that all students can see the performance of the trick as much as possible, for example, if the teacher sits on the floor and the students gather around. To perform this trick, the teacher needs to select 32 cards from the deck with which the trick will be performed from the entire deck of cards (if there are no ready-made decks of cards in the class, the trick can also be performed with 16 cards).

It is necessary to tell the story of a pirate who wants to hide a treasure and his enemies who follow the pirate and try to find out where the treasure is hidden in order to take it for themselves. It must be clear that the pirate wants to hide the treasure very well. Students observe the trick.

This part of the activity usually results in student delight, disbelief, and questions like "How did you do that?" The teacher asks the students if they want to know how and why the trick works and if they want to learn how to do the trick.

The teacher repeats the trick once more, to show that the success in the first performance was not accidental.

PART 2: Disclosure and explanation of the trick (40 minutes)

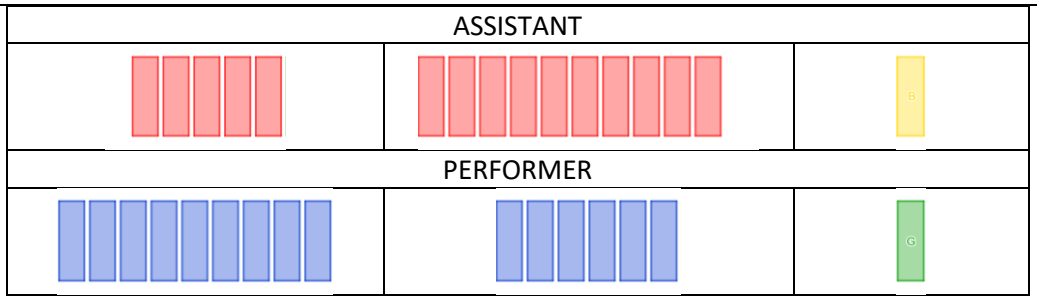
For math tricks, this is a key part because the goal is for students to understand the math background of the trick, i.e., why it works, because only then will they understand that the trick is not based on fraud but on math, i.e., science, and that there is a logical explanation for why the trick works.

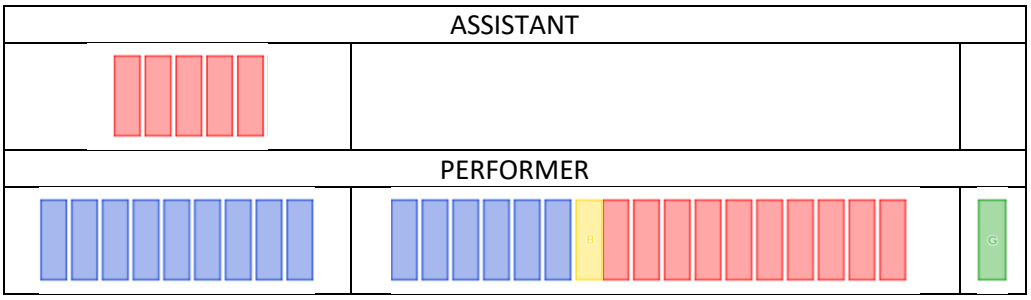
Students sit in pairs, and each pair receives a deck of cards. Each pair determines the student who will be the performer and the student who will be the assistant (in the first performance of the trick).

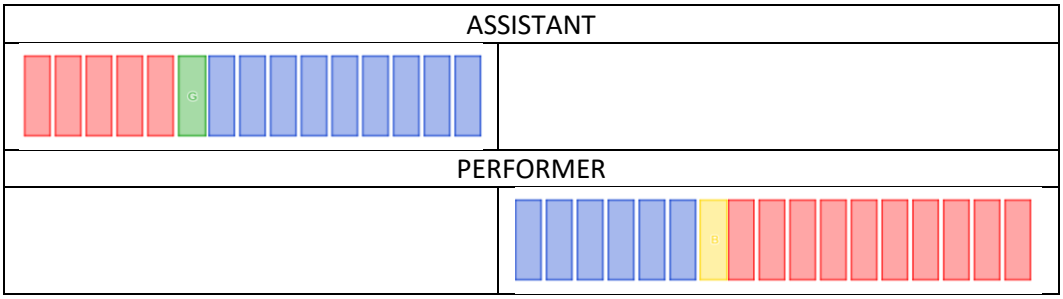
Each pair listens carefully to the teacher and works according to the instructions.

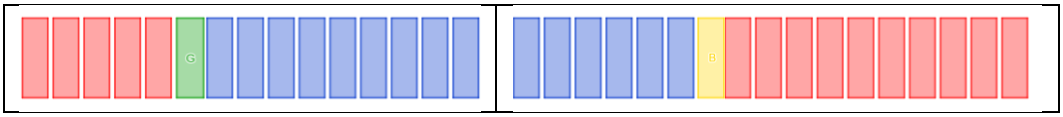
1. Students count the cards in the deck. They conclude that there are 32 cards.
2. Two special cards representing the pirate and his treasure are drawn from the deck (e.g. the pirate can be represented by a king and the treasure by an ace of the same colour). The students conclude that there are now 30 cards in the deck.
3. They place the pirate and treasure face up and divide the remaining cards into two decks. Students conclude that each deck has 15 cards because (32-2):2=15. One deck remains with the performer, and the other is taken by the assistant. Also, the performer takes the pirate card, and the assistant takes the treasure card.
4. The performer cuts the closest deck in two (whatever) and asks the assistant to do the same with the deck closest to him/her.
5. Now it's time to hide the treasure. The performer asks the assistant to place the treasure face up (so that we can follow it while we learn the trick) on one of the two decks made by the performer and to cover the deck on which he placed the treasure with one of his decks (whatever).
6. The performer takes the pirate and places it face up on the deck left in front of the assistant. The performer takes his "minor" deck (the deck that does not contain treasure) and covers the pirate. Now there are two decks on the table that the performer puts on top of each other.

At this moment, the teacher stops the trick and together with the students, analyses how the cards are arranged in the deck. Let's observe what happens in the process of hiding the treasure in the deck







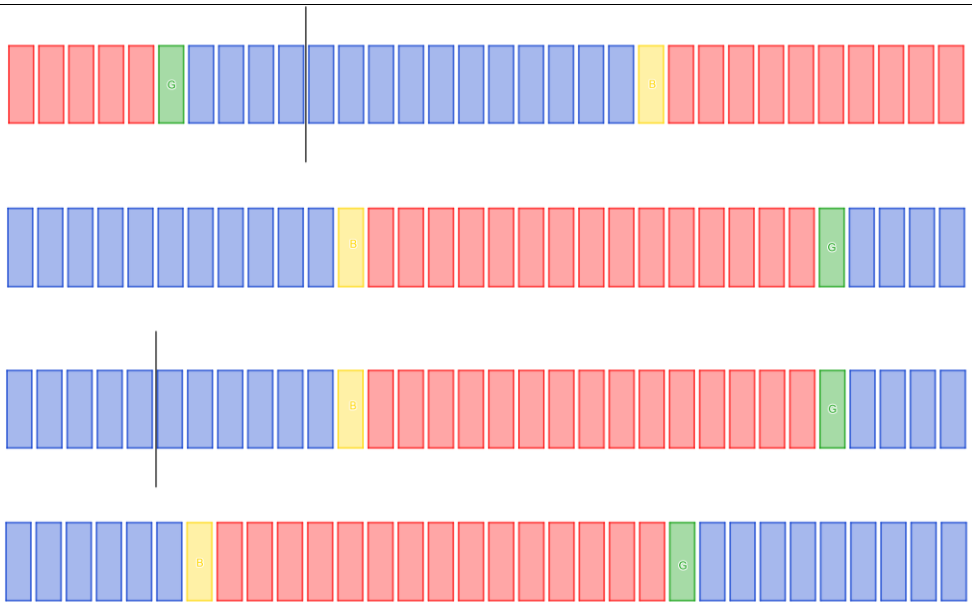


After overlapping those two suits on top of each other (it does not matter which order) we have 32 cards arranged as follows: 5, G, 9, 6, B, 10 or 6, B, 10, 5, G, 9.
We notice that between the pirate and the treasure ("inside") there is a connected pile that we created in step 3 (a pile of 15 cards). Cards from the second pile of 15 cards are located "outside", that is, at the beginning and end of the deck.

7. The teacher asks all students if the treasure is well hidden or if it is lost in this deck? To make sure that it is well hidden, the performer asks the assistant to cut the deck in two and place the two pieces on top of each other, reversing their order.

The teacher pauses again and tries to explain why cutting the deck will not change the number of cards between the pirate and the treasure.

If we cut the deck, will there still be 15 cards between the pirate and the treasure?



8. The teacher deals the cards alternately into two piles (1st card on the left, 2nd card on the right, 3rd card on the left, ...) until they run out of cards.

The teacher pauses the performance of the trick again and asks the following questions are the pirate and the treasure in the same deck (students who have performed the procedure correctly will have the pirate and the treasure in the same deck, and the teacher helps those who do not have the pirate and the treasure). Students notice that the deck containing the pirate and the treasure contains 16 cards and that there are now 7 cards between the pirate and the treasure. The following is an explanation of why the pirate and the treasure are in the same deck and why there are now 7 cards between the pirate and the treasure.

We divide the entire deck into two piles of equal numbers like this: we alternately sort the cards one by one into two piles: left and right. The teacher emphasizes that in the left pile there will be cards in odd positions, and in the right pile there will be cards in even positions. The teacher repeats with the students what even and odd numbers are. We write on the board:

1	2
3	4
5	6
7	8
9	10
...	…
31	32

We notice that after the procedure is completed, we will have the same number of cards in each pile: 32/2=16.
How are we sure that the pirate and the treasure are in the same pile?

Let us imagine that the pirate was the 7th card. After the pirate we have 15 more cards

and the 16th card after him is the treasure. Students count: 7+16=23, the treasure is the 23rd card. We see that then both the pirate and the treasure cards are in an odd position, so we know that they are in the left pile. The teacher shows the separation procedure on the board.

Let's notice that in the next step, after discarding the pile in which there is no pirate and treasure, we will have 7 cards between the pirate and the treasure, that is, the treasure is the 8th card after the pirate because 16:2=8 (we discarded every other card). Now the deck has 16 cards and is (16-2):2=7.
The teacher emphasizes that in this example we added the odd number (7) to the even number (16) and got an odd number (23) and asks if it will always be like that. Is the sum of an odd number and an even number always an odd number? Students remember what they learned and conclude that the answer is yes.

The teacher asks the following question. What would happen if the pirate was in an even position? Where would the treasure be then? Students remember that the sum of two even numbers is an even number and conclude that the treasure will also be in an even position. If necessary, the teacher writes again on the board all the numbers from 1 to 32 in two columns and explains the situation.
Let's notice the regularity: in the first step we have a total of 32 cards and the treasure is the 16th card after the pirates.

In the second step, we have a total of 16 cards in the pile and the treasure is the 8th card after the pirate.
In the third step, we will have a total of 8 cards in one pile and the treasure will be the 4th card after the pirates, ...

9. The performer discards the deck that does not contain the pirate and the treasure and continues only with the one that contains the pirate (which has 16 cards).
10. The performer again divides the deck into two decks (1st card on the left, 2nd card on the right, 3rd card on the left, ...). The students conclude that the pirate and the treasure are in the same deck again. Furthermore, they conclude that that deck has 8 cards because 16:2=8 and that there are exactly 3 cards between the pirate and the treasure because (8-2):2=3.
11. The performer again divides the deck into two decks (1st card on the left, 2nd card on the right, 3rd card on the left, ...). The students

	conclude that the pirate and the treasure are again in the same deck. Furthermore, they conclude that this deck has 8:2=4 cards and that between the pirate and the treasure is exactly (4-2):2=1 card. 12. In the last step, the performer again divides the deck into two decks, each deck has two cards, and once again the pirate and the treasure are in the same deck, i.e. the pirate has found his treasure. 13. From the above procedure it is now clear that the trick can be performed using 4, 8, 16, 32, 64, 128... cards. PART 3: Practicing the trick (25 minutes) Now the activity is transferred to students and they try to repeat the trick, following the procedure the teacher showed them. Students, working in pairs, repeat the trick several times. If necessary, the first few times can be performed simultaneously by all pairs so that the teacher can make sure that everyone can do the trick. For homework, students practice the trick and present it to their housemates (students are given written instructions for performing the trick). In addition, students receive the following assignment (and worksheet) as preparation for the next lesson: • imagine a pirate, describe him, and draw him, • imagine the pirate's treasure, describe it and draw it (try to think of some reason the pirate wants to hide the treasure), • imagine another character who wants to find the treasure, describe him, and draw him (try to think of a reason he wants the treasure). ADDITIONAL ACTIVITY (45 minutes): The story of the pirate and the treasure In the first part of the activity, the students perform the trick they practiced. After practicing the trick, the teacher, and the students together, with the help of a worksheet, devise and write down a story that will accompany the trick (performed by two students). As the flow of the trick directs the story, the story will not be the same every time it is performed. Students write finished parts of the story in the scheme and independently, while performing the trick again, fill in the scheme and produce different variants of the story (depending on the course of the trick).
Extension activities	4. Designing and/or finding music and scenery to perform the trick. 5. Rehearsal and performance for a school activity. 6. Connecting with the contents of science, students can be given tasks to create a story that will include places and regions that they mentioned in science lessons.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Math card tricks: Clock**
Key words	math card trick, counting to 20, clock with hands
Short description	The activity is based on math card tricks which develop a positive attitude towards math in students. This activity is an excellent way to strengthen pre-math and math skills in a fun and creative way that increases students' interests and motivation. Math background of the Clock trick involves counting to 20 and knowing how a clock with hands works. The activity develops students' motor skills, the ability to follow and reproduce the procedure and the focused implementation of the procedure according to exactly given instructions, in which the final success depends on the successful implementation of each individual step. It is possible to expand the activity by creatively designing performances within mother tongue, foreign language, art, and music classes.
IT tools	
Fields (select)	A1: **Math** A2: **Natural science** A3: Art A4: Music A5: **PE** A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: pre-math and math skills: repetition of the procedure in an exact order; units and tens; counting to 20
	A2: time and analog clock
	A5: motor skills and fine motor skills in hands
	A6: creating a story; communication with the audience; giving clear instructions
Expected prior knowledge	counting to 20; analog clock
Expected learning outcomes	• Uses natural numbers up to 100 to describe and display quantity and order. • Adds and subtracts in a set of natural numbers up to 100. • The student explains the organization of time and displays the timeline of events. • He actively participates in elementary games that develop self-esteem, self-confidence and perseverance. • The student reads/listens to short texts thematically appropriate to the student's experience, language development and interests • The student talks and speaks in accordance with the theme of everyday life and respects the rules of polite conduct. • The student finds the data in the read text according to instructions or questions. • The student creatively expresses himself according to his own interest motivated by different experiences and experiences of literary text

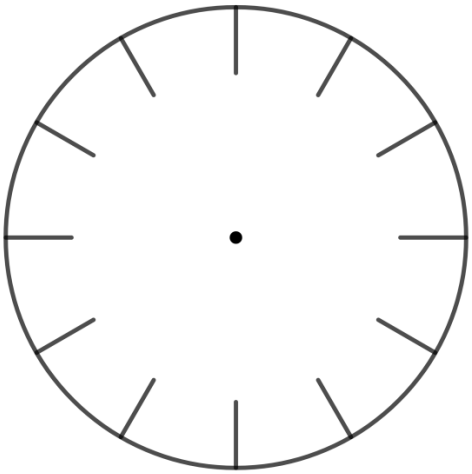
Expected duration	90 minutes
Preparation	8. Prepare decks of cards (one deck per two students) 9. The teacher practices the trick (according to the instructions: (https://inamath.uniri.hr/wp-content/uploads/2022/11/Math-card-trick-Clock-Instructions.docx). 10. The teacher learns the math behind the trick
Detailed description of activities	PART 1: Presentation of the trick by the teacher (15 minutes) The teacher announces that a math card trick will be performed. To perform the trick, it is not necessary to have dexterous hands, but to let the math do the magic part. At the beginning of the presentation, the teacher explains what a math card trick means: in math card tricks, the trick relies exclusively on math, without the necessary skills of the trick performer, without "fake" and marked cards and hidden information. In the introductory part, it is emphasized that the most important part of today's activity is the following. • Students listen carefully to the teacher and follow the instructions. • Students carefully, slowly, and concentratedly carry out everything the teacher tells them. • Students do not play with the cards but stop after each individual step and wait for further instructions. To perform this trick, the teacher needs one deck of cards, which must have at least 24 cards. The teacher performs the trick several times as instructed without any further explanation. Students observe the trick. It is important that the teacher and students are positioned so that all students can see the trick, for example, if the teacher sits on the floor and the students gather around him. This part of the activity usually results in student delight, disbelief, and questions like "How did you do that?" The teacher asks the students if they want to know how and why the trick works and if they want to learn how to do the trick. PART 2: Disclosure and explanation of the trick (45 minutes) For math tricks, this is a key part because the goal is for students to understand the math background of the trick, i.e. why it works, because only then will they understand that the trick is not based on fraud but on math, i.e., science, and that there is a logical explanation for why the trick works. In this part, the teacher repeats the trick and explains the trick, and the students make sure that the explanation is correct by counting the cards and following the corresponding card. The performer separates part of the cards (13) from the deck, gives them to the assistant and asks to shuffle the cards given. A particularly important part of the trick happens in this step. This part of the trick reveals a lot, so it is often not mentioned aloud that exactly 13 cards have been dealt. 1. The performer asks the assistant to cut their deck of cards and separate one part from the side (blue cards).

Two piles of assistant cards have a total of 13 cards.
The assistant should once again shuffle the cards in their hands (red cards) and remember the bottom card.

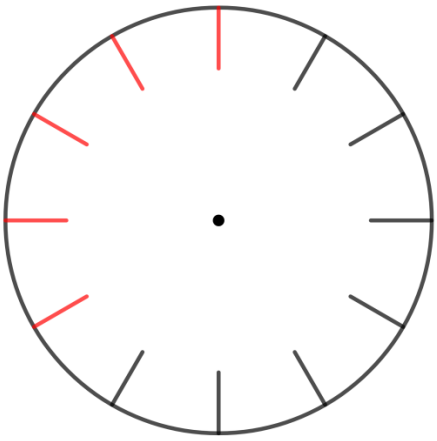
2. The performer puts the cards that were with the assistant (red cards) on top of his deck.

To explain more easily, let's mark the number of cards in the retained pile (blue cards) with 13-X and with X the number of cards in the pile with the memorized card (red cards).
The pile of cards with the memorized card (red cards) is placed on top of the other cards, so we know exactly where the memorized card is (Xth card in order).

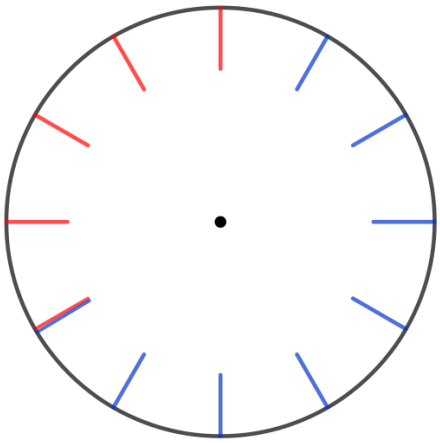
The performer deals the cards face down from the top of the deck and places them in a clock shape starting at 12 o'clock.



3. Cards are dealt counter clockwise.



4. When the cards have been dealt, the performer asks the assistant to count the pile of cards that he/she separated at the beginning.

	 When we deal the cards in the shape of a clock (starting from 12 counter-clockwise), the memorized card is in that case the X card if we count from 12 counter-clockwise, that is (12+1-X) th card if we start from the card in the place of number 1. Note that exactly 13 cards are needed at the beginning of the trick because we count the memorized card twice: the first time when we count counter clockwise from 12 to the memorized card and the second time when we count clockwise from 1 to the memorized card. 5. The assistant should then look at the card that is on the same clock hour as he counted cards. The turned over card is exactly the card that the assistant has memorized. PART 3: Practicing the trick (30 minutes) Now the activity is done by students and they try to repeat the trick, following the procedure the teacher showed them. The students, working in pairs, repeat the trick until they practice it and learn to perform it without a mistake. It is essential that at least once all pairs of students perform the trick simultaneously (all pairs for the same number), following the teacher's instructions, and that the student controls the performance. It is essential that at least once all pairs of students perform the trick simultaneously (all pairs for the same number), following the teacher's instructions, and that the student controls the performance. During that time, the teacher once again repeats the explained trick and the students make sure that the explanation is correct. For homework students can practice the trick and present it to their housemates (students are given written instructions for performing the trick).
Extension activities	7. Telling a story and details that will make the trick interesting to the audience. 8. Designing and/or finding music and scenery to perform the trick. 9. Rehearsal and performance. Each trick is a small performance that students must practice and perform in front of an audience. The trick requires the students to talk to the audience, give clear instructions, and lead the audience through the story they have created so that the trick is more than just a mathematical procedure.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Department of Mathematics, University of Rijeka)

Title	**Healthy lifestyle**
Key words	health, lifestyle, physical activity, healthy eating, collecting and organising data
Short description	In this unit students: •learn about the importance of a healthy lifestyle (adequate fitness, physical care, healthy eating, stress relief, and relaxation, balance between learning, physical activity, rest, and sleep), •solve a problem that requires collecting and organising data, presenting it clearly, as well as reading and interpreting it, •present the data in a chart, table, etc.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: Mother tongue A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: problem-solving, collecting and organising data
	A2: healthy lifestyle (eating, exercise, rest…)
	A5: adequate fitness for a healthy lifestyle
	A7: English terms to describe healthy lifestyles
Expected prior knowledge	Counting to 20, name of days, the food pyramid
Expected learning outcomes	LEARNING OUTCOMES - MATHEMATICS • to solve a problem that requires collecting and organising data, presenting it clearly, as well as reading and interpreting it; • present the data in a chart, table, etc. LEARNING OUTCOMES – NATURAL SCIENCE • to know that healthy eating, exercise, and rest enable growth and development and help them stay healthy; • to create and complete tables; create tables from data, and draw bar charts from tables; LEARNING OUTCOMES – SPORT • to learn about the importance of a healthy lifestyle (adequate fitness, physical care, healthy eating, stress relief, and relaxation, balance between learning, physical activity, rest, and sleep) LEARNING OUTCOMES – ENGLISH • to recognise English terms to describe healthy lifestyles and can determine what is healthy or less healthy (by using the terms 'It's healthy.'/'It's not so healthy.')
Expected duration	90 minutes
Preparation	food pyramid, drawing paper, flyers (advertising leaflets), worksheets (appendices)
Detailed description of activities	**INTRODUCTORY PART/WARM-UP** Open the window to let fresh air into the classroom. 1. First, do some movement activities with the learners (each learner finds his/her own space and, according to the teacher's instructions, does 5 jumps, 5 forward bends, 5 squats, 5 steps to the chair, sits down on the chair, sits still

	and counts to 10 in silence, then opens his/her eyes and waits for further instructions). After the exercises, we discuss the importance of movement for our health. 2. Show the PPT with some food-related proverbs and discuss their meanings, e.g. "An apple a day keeps the doctor away." "Hunger is the best cook." “Eat breakfast like a king; lunch like a prince; dinner like a pauper.” etc. **MAIN PART** Together with the learners, we revise what a healthy, balanced diet is (using the food pyramid). Learners try to recall what they ate yesterday that was healthy, what they ate yesterday that was less healthy, and what else would be good for them to have eaten. They think about how many meals they had. We also mention that it is important not to eat in front of the TV or in front of the computer, phone, etc. We talk about the importance of fluid for our bodies, what is healthy to drink and how much. After the discussion, we invite the learners to drink some water. We discuss with the learners what else, besides exercise and a healthy diet, is important in order to maintain a healthy body. Rest: learners tell us when they went to bed last night and how they feel at the moment - are they sleepy, tired, ... Together we calculate how much sleep a learner who went to bed at 9pm and got up at 7am, for example, got. We tell them that it is essential for the health of children their age to sleep at least 10 hours a day. Hygiene: we talk about how we take care of our hygiene. We revise how to wash our hands and brush our teeth properly and how to take care of the hygiene of our whole body. We divide the learners into groups and each group makes a poster presenting a healthy lifestyle (they draw or stick out what we need in order to stay healthy - they can cut it out of magazines, advertising leaflets, etc.). **CONCLUSION** We explain the learners the weekly task of keeping track of healthy lifestyle habits, which will be recorded in the tables (Appendix 1). In a week's time, we'll talk about the tables: On which days did you sleep at least 10 hours? On which occasions have you washed your hands? Did you do it only once a day? In which parts of the day did you brush your teeth? Did you exercise every day in the afternoon? On which days did you exercise in the afternoon? Have you been eating mostly healthy all the time? What could you change in your diet to make it even healthier? Which liquid is healthy for the body? Do you think you have drunk enough

	fluids in the past week? Give learners a table to enter the total number of glasses per day (Appendix 2) - students fill in the table by colouring the number of glasses of liquid for each day. Then we focus on the interpretation. How many glasses of liquid did you drink on Monday? On which day did you drink the most/least fluids? Why do you think you drank less/more fluid on a particular day? How many glasses of liquid did you drink in total (in the whole week)? What do you like to drink? We explain to the learners how to represent the number of glasses of liquid in a bar chart (the meaning of the legend). **THE ENGLISH LESSON (CLIL)** In English, learners learn about the food pyramid and how to tell which foods are healthy and which are not so healthy, and later use pictures to similarly classify other habits[2]. As an introduction, the teacher can play (or read) "The Very Hungry Caterpillar": https://www.youtube.com/watch?v=75NQK-Sm1YY They then discuss which types of the food the caterpillar has eaten is healthier. The teacher shows them the food pyramid and together they repeat the food terms and say which foods are healthier and which are less healthy. They can listen to/sing the song "This is the way..." (https://www.youtube.com/watch?v=zoJjUHBNufY). The teacher can ask the learners, "What do we do to stay healthy?" ...We wash our face. We comb our hair. / We brush our teeth. / Wash our hands. She/he provides enough scaffolding (pictures, gestures) at all times. Finally, the teacher hands out pictures with healthy/less healthy habits (including food), e.g. washing hands, dirty hands, brushing teeth, exercise, sitting in front of the TV, lollipops, carrots, water, fizzy drinks, etc. In pairs, the learners arrange the pictures in a table with two columns (healthy/not so healthy). The teacher then shows the different pictures and asks e.g. 'Is this healthy?' and the learners answer 'It's healthy. /It's not so healthy.' (If learners already know the vocabulary in the pictures, the teacher can use the terms instead of the pictures, e.g. 'Is washing our hands healthy?')
Extension activities	Using prepared materials, similar activities can be carried out in higher grades.
Additional notes	Appendix 1 and 2
Authors	Marina Volk, Tadeja Volmut, Mojca Žefran, Nataša Dolenc Orbanić

[2] If the learners already know the food vocabulary, healthy habits can be added later and classified, otherwise the diet/food is discussed in more detail and then the learners classify it as more or less healthy.

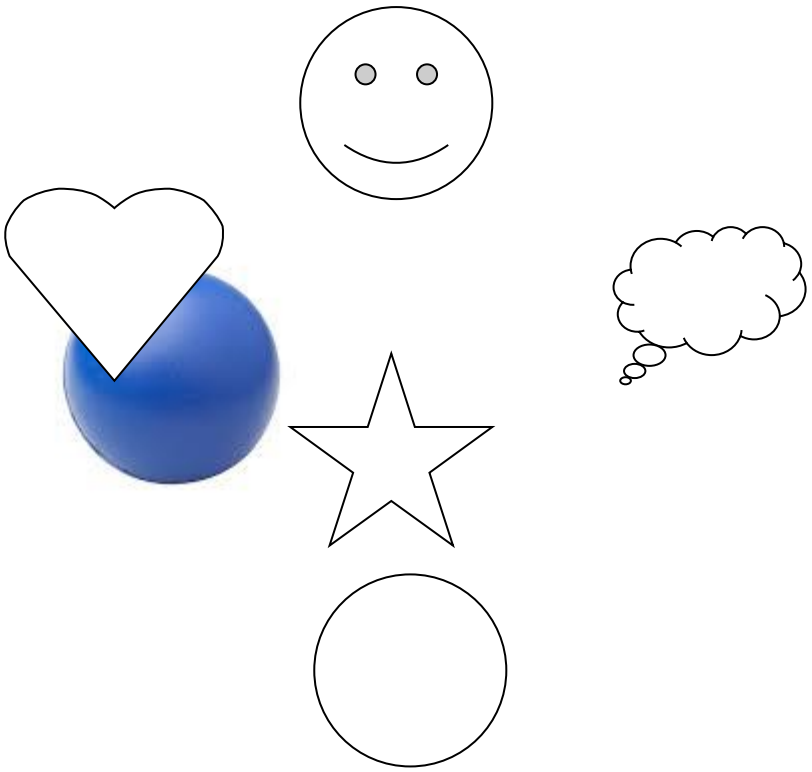
If healthy habits are also added, have the pupils learn the English terms for the habits in the habit tracking table (Appendix 1) so that they can also report in English (the teacher estimates/decides if the pupils would be able to do this).

Title	**Classification**
Key words	Classification, identification, animals
Short description	In this unit students: •classify according to one or two characteristics using the tree diagram and Carroll diagram, •identify and articulate the property by which the elements have been classified, •display and read/identify the classification of elements in the tree and Carroll diagrams. Students classify living things (plants and animals) and use English names (movement is also included).
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: Mother tongue A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: classify according to one or two characteristics using the tree diagram and Carroll diagram
	A2: identify, name and compare different life forms
	A5: different natural forms of movement
	A7: name different animals in English
Expected prior knowledge	Classify to one characteristic, diagrams
Expected learning outcomes	Learning outcomes - MATHEMATICS • to classify according to one or two characteristics using the tree diagram and Carroll diagram, • to identify and articulate the property by which the elements have been classified, • to display and read/identify the classification of elements in the tree and Carroll diagrams. Learning outcomes – NATURAL SCIENCES • to identify, name and compare different life forms. Learning outcomes – ENGLISH • to name different animals in English, • to follow instructions in English by responding to them with movement. Learning outcomes – SPORT • to revise different natural forms of movement (different forms of walking, running, jumping and crawling).
Expected duration	90 minutes
Preparation	worksheet for animal classification, large pictures of animals, poster with a tree diagram, different tree leaves, worksheet with a song

Detailed description of activities	**INTRODUCTORY PART/WARM-UP** We divide the space in front of the board into two parts, using mason's tape or a skipping rope (you can also do this in the corridor or in another room where there is enough space). We show the learners the picture of the blue trousers and the picture where the blue trousers are crossed out. We discuss what the two pictures show. We tell the pupils that today we are going to use pictures like these, which show certain characteristics, to classify things. We ask the pupils to stand in a semicircle in front of the board and then we place the picture of the blue trousers over the left part of the "Carroll diagram" in front of the board and the picture of the crossed-out blue trousers over the right part. I ask each student where they would place themselves in the display according to the colour of the trousers they are wearing. Then all the learners line up in the Carroll's diagram in front of the board. We lead the discussion: How many learners are wearing blue trousers today? How many learners are not wearing blue trousers today? What colour of trousers are the learners who are not in the group of pupils with blue trousers wearing? Which learners are wearing blue trousers today? ... **MAIN PART** All the learners step out of the diagram and we show a picture with a new characteristic, e.g. a boy and a crossed-out boy. The pupils are classified by stepping into the diagram according to whether they are boys or not. They then step out of the diagram (the pictures of the boy, not-boy characteristics still remain) and we halve the diagram to create 4 spaces to classify by two traits at the same time. we also show the learners a picture illustrating that a pupil has walked to school today and the negation of this characteristic. We discuss what the picture shows, then we put both pictures in the diagram. We ask the learners where they would classify a learner who is a boy and who has walked to school. We direct the learner to the appropriate space in the diagram. Then the other learners are classified with the help of the teacher. Once all the learners have been classified, we lead the discussion: How many boys did not walk to school today? How many of the non-boy pupils walked to school? Which group has the most children? Which characteristics does this group have? etc. Students take their seats and we project a picture of a Carroll diagram and a group of animals on the interactive whiteboard. The learners are given the same diagram and pictures of animals on a worksheet (Appendix 1). We discuss what the pictures, according to which we will sort the animals, show: first characteristic: flies - does not fly, second characteristic: lives in the forest - does not live in the forest. We help the learners classify each animal in the Carroll diagram and discuss the animals briefly. We ask the learners questions about each animal (e.g. Which animal is in the picture? Where does it live? How does it move? Describe its structure. What are its special features?). We introduce a second diagram that can be used for classification – the tree diagram. We attach the tree diagram (shown on the poster, Appendix 2) to the board and tell the learners that we are going to classify the leaves of the

	trees. We put the different tree leaves (e.g. oak, chestnut, maple, lime, beech, etc.) in front of the board and each learner chooses one and classifies it using the poster. We show them the characteristics by which we will classify the leaves, e.g. according to the shape of the leaf (egg-shaped, not egg-shaped) and according to the leaf edge (toothed/serrated, lobed/projection, entire/smooth). We classify the first leaf ourselves, indicating the path along the trunk and branches into the appropriate canopy according to the leaf characteristics. Then, together with the learners, we find out which plant the leaf belongs to and we discuss each plant (e.g. fruit, trunk, shape of the canopy, etc.). **CONCLUSION** English and sport lesson (CLIL): We sing the song "I can..."(Appendix 3) and invite the learners to sing and dance with us. We put the animal pictures on the board and ask the pupils: 'What's this? If the pupils don't know the term, we name it, e.g.: This is a dolphin. For each animal, we first check with the pupils if they can name it; if not, we name it ourselves. We include the following vocabulary items: a dolphin, a fox, a squirrel, a bear, a snake, an owl, a swan. Then we take all the pictures and flash one of the pictures (show really shortly and then hide it) to the learners; they try to work out what was in the picture. Then we change the game a little and reveal the animal picture very slowly bit by bit, and the pupils have to find out what is in the picture as quickly as possible. We make space with the students in the classroom. We sing the song "I can..." again and dance while singing. Then we say to them, "Walk like a bear". The pupils follow the instructions and respond with the appropriate movement. Other instructions: Run like a fox. Crawl like a snake. Fly like a swan. Swim like a dolphin. we include movement activities and invite pupils to also give instructions to each other In the next part of the lesson, the pupils can learn about the structures can/can't from the song (we use pictures to show the difference) and can later classify the animals in English (can/can't fly).
Extension activities	Using prepared materials, similar activities can be carried out in higher grades.
Additional notes	Appendix 1-3
Authors	Marina Volk, Tadeja Volmut, Mojca Žefran, Nataša Dolenc Orbanić

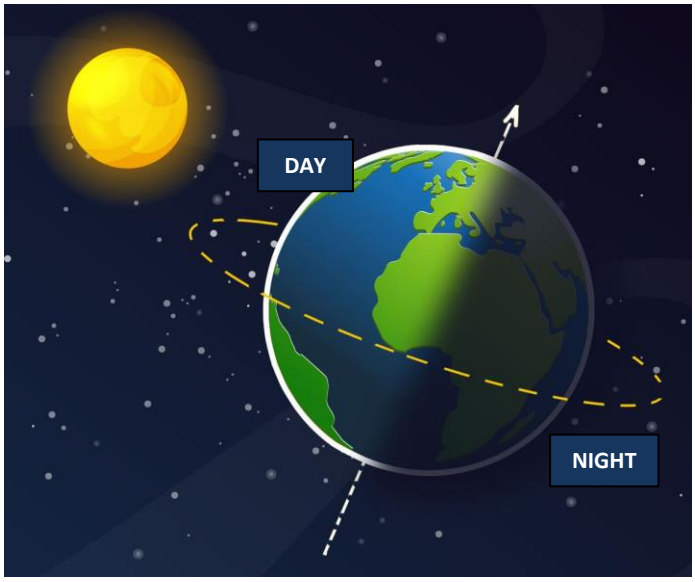
Title	**Where am I?**
Keywords	Position of objects, relationships in space
Short description	As part of this activity, students get to know the concepts of road, street, sidewalk and pavement. They will learn to determine the position of objects in space. The students' task is to describe their way from home to school using the relationships up-down, left-right, front-back, front-back.
ICT tools included	Interactive games
Areas (select)	**A1: Mathematics** **A2: Natural science** **A3: Art culture** A4: Musical culture **A5: Physical culture** A6: Mother tongue A7: Foreign language Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: relationships in space
	A2: finding your way in space
	A3: drawing, coloring
	A5: day-night game
Expected prior knowledge of students	Basics of spatial orientation
Expected outcomes	Recognizing the position of objects and relationships in space
Expected duration of the activity	45 minutes
Preparation of activities	Prepare tasks for determining the position of objects.
Detailed description of all teaching activities	At the beginning of the activity, the teacher asks the students questions: *Describe where you sit in the classroom!* *Which hand do you write with?* *Where is the sun in relation to you?* The teacher connects the lesson in Natural Science: *Navigating in space; Traffic - road and street* with a Math lesson *Location of the subject.* Students get to know the concepts and appearance of roads, streets, roadways and sidewalks. The teacher explains that the road is a thoroughfare outside the settlement, that it has a carriageway, drivers drive vehicles on the carriageway and pedestrians walk along the side of the road. The teacher explains that the street is a thoroughfare in the settlement, which consists of sidewalks and roadways, and that cars drive on the right side of the roadway. In the second parto f the activity the task is to describe journey from home to school using up-down relationships front-behind back and forth left right.

	After that, the students should relate the given concepts to the blue ball using the following relationships: up and down front-behind left right.  In the thirt parto f thw activity, on blank papers, students should draw a boy looking towards the student, and mark his left and right hand. In the final part of the activity, the students play the game "Day-Night", where day means up and night means down.
Possibilities to expand activities	It is possible to create online interactive games with spatial suggestions, such as Memory, Find a pair.
Additional notes	
Authors	Antea Čilić, Hrvoje Ljubić, Mila Zovko (FPMOZ, University of Mostar)

Title	**How long is a year?**
Key words	time, day, month, year, seasons, Sun, Earth
Short description	The activities begin by explaining the concept of day by rotating the Earth around its axis, then by clarifying day and night, and finally by using the week from Monday to Sunday as an example. The months and seasons are then explained to the students, and the start of each season is marked on the calendar. This reminds students of the calendar and connects it to the seasons, different dates such as birthdays, the start of school, vacations, and so on. Finally, the rotation of the Earth around the Sun is used to explain the year to the students. Students can be placed in the positions of the Sun and the Earth, so that as they turn, they mark the changes of days and seasons in the classroom. Students are given material in which they are asked to mark specific dates for each month and draw what they associate with that date. One aspect of the activity can be completed on the computer using the Paint program.
ICT tools included	*Paint programme*
Fields/ subjects	A1: Mathematics A2: World around us *A3: Art (indirectly)* *A6: Serbian language or any other (indirectly)*
Topics (for each selected subject)	A1: Measurement and measures: Time measures – hour and minute; Time measures - day, week, month and year
	A2: Day. Week. Month and year. Calendar. Seasons
	A3: indirectly involved - drawing, coloring, filling in fields
	A6: indirectly involved - writing numbers, dates, simple sentences
Expected prior knowledge of students	☞ Recognition, reading and writing the numbers and dates ☞ Recognition of time measures ☞ Elementary knowledge about seasons and alternation of day and night ☞ *Elementary skills and knowledge about colours, shades, drawing and colouring* ☞ *Elementary knowledge and skills of writing numbers, dates, simple sentences* ☞ *Basic computer skills*
Expected outcomes	☞ Improving the knowledge about the time and time measurement ☞ Identifying the relationship between the changing time and the Earth's rotation ☞ Identifying the relationship between seasonal changes and the Earth's revolution
Expected duration of the activity	2 school hours (90 minutes), with the possibility of extension to additional Art and Serbian language classes (any other language)
Preparation of activities	Download the Power Point Presentation of the scenario and all the exercise documents via link https://inamath.uniri.hr/how-long-is-a-year/
Detailed description of all teaching activities	**Activity 1: F Improving the knowledge about the time, time measurement and writing the time** The teacher repeats the determination of the time (day, hour, minute) with the students. She/He calls several students and asks them to answer about what time it is now, what time they got up, what day of the week it is, how many days are left until the weekend? How many hours are there in a day, how many days in a week, how many days in June? Then she/he invites them to discuss the current season, what will happen next, what came before, and so forth. Please raise your hands if you were born in the spring! To verify that they have answered correctly and perform other comparable tasks, the teacher calls several students to check. **Activity 2: Determination and improving the knowledge about Earth's rotation**

The teacher continues on schedule with a story about the Earth's rotation around its axis and around the Sun. So it was daytime when you woke up this morning. It was night when you went to bed last night. Why do the day and night alternate? Earth is one of the solar system's planets. It is constantly rotating around itself, i.e. around its axis. It happens slowly enough that we don't notice it while we're moving, but we do notice it in another way. The earth rotates once every 24 hours. As it rotates, one part of the Earth smiles at the Sun, while the other part of the Earth sleeps and there is night.

Let's pretend we're the Earth and the Sun. The teacher animates two students into the Sun and the Earth and explains how change occurs. Milan, for example, represents the Sun, while Ana represents the Earth. Ana spins in a circle around herself and when Ana sees Milan face to face, it'a daytime. When Ana slowly turns around and no longer sees Milan, it's time to sleep, and it's getting dark, night is coming.



On the board is a sketch of the Earth's rotation on its axis. The axis is the white dotted line that runs through our planet's center. The one half is always facing the Sun, while the other half is always in the dark. As the Earth moves towards the Sun, dawn begins, the day begins, and it is time to wake up. Your peers are going to bed on the other side of the world. So it happens every day, and a day, as we all know, is 24 hours long. A week is made up of seven days. We go to school for five days and then take two days off.

Activity 1.1: The days of the week - exercise

The teacher hands out to the students a table with some of the weekdays filled in. The remaining days are filled in by the students themselves, and they are instructed to color each day a different color and write a sentence that is related to that day. This element of the activity has a tenuous connection to Serbian language (any other) and art.

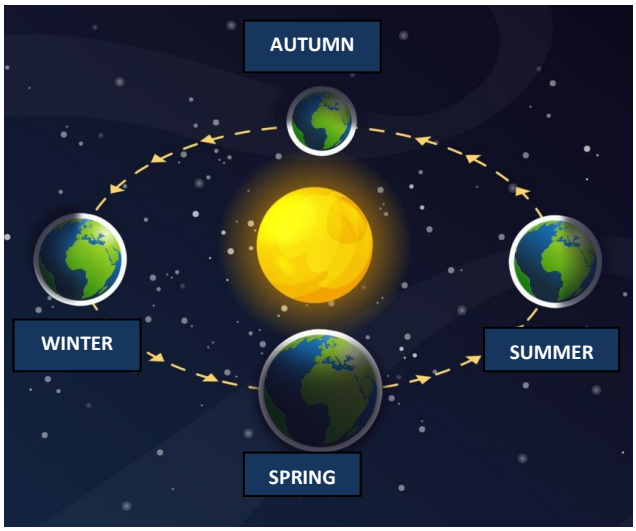
Weekdays table (download a printable version):

Monday		Wednesday		Friday		Sunday

Activuty 3: Determination and improving the knowledge about Earth's revolution

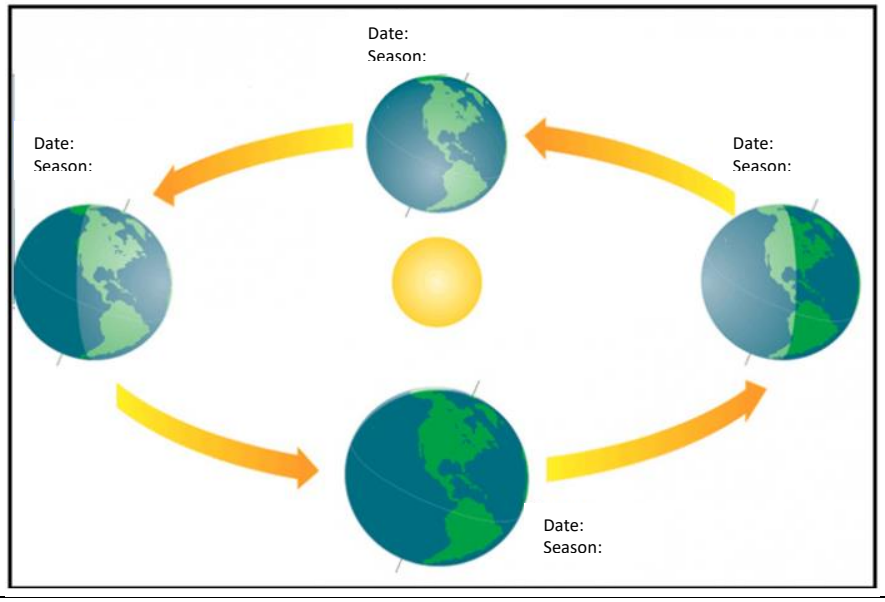
The teacher and the students continue to talk about the weather. So, the Earth goes around itself once every 24 hours. We remember it like night and daytime. Why do things like rain, snow, heat, and wind change around us? Does it snow all the time? No, it doesn't fall all the time. That time is the winter season. What are the other seasons? Why do they alternate? The Earth doesn't just spin around itself; it also moves in its own way around the Sun. This path looks most like a stretched circle and it takes one year for the Earth to go around this circle. There are 12 months in a year, and every three months, a new season starts. When the Earth is farthest from the Sun, it doesn't warm us as much, so winter comes. Summer is when the Earth is on its path closest to the Sun. What seasons are in between?

Activity 3.1. The Erath's revolution – exercise



The teacher can put the students in the position of the Earth and the Sun, draw the path of the Earth, and mark the seasons as the students move along that path.

The teacher then gives each student a sketch and tells them to write on it when each season starts.



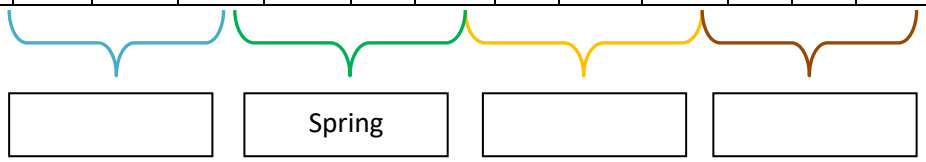
When does the next season start, and what is it? How do we figure it out if today is the 30th of May (which could be different depending on how the scenario is played out right now)? May has 31 days. Next month is June, and summer starts on June 21. So, one more day in May and twenty days in June make a total of twenty-one days until summer starts. Each season has its own beauty and is different from the others.

Activity 4: The exercise - calendar

The teacher gives the students a table with months to do in class or as homework. The task is to write the months, seasons, and important dates (birthdays, holidays, etc.) for each month, draw something that reminds them of that month, and write one sentence about that month (or date). This activity has a link to art and the Serbian language (any other).

Table - calendar (download the larger one that can be printed on one A4 sheet and distributed as such to the students; here it is written abbreviated due to space):

	Jan	Febr	Mar	April		Jun		Aug	Sept			Dec
Drawing												
Date												
Sentence												



Possibilities for expansion of activities	**Possible activities related to Art and Serbian language (any other language)** Aside from the indirect link to art that was already mentioned, students can also draw pictures that have something to do with a certain season. They can also be asked to write as many words or sentences as they can think of that relate to each month. For example, for June, they might write six words or sentences.
Additional notes	If a student attends classes according to the IOP1 or IOP2 program, it is necessary to include a personal companion/assistant in the preparation, and choose activities in which it is possible to include the student as much as possible.
Authors	**Milica Solarević, PhD, Associate Professor, UNS**

Title	**Jumping Math**
Keywords	Natural numbers up to 100 Multiplication and division Change of seasons and duration
Short description	Connecting physical education, arithmetic operations up to 100, and science (knowledge of the change of seasons). Children are divided into teams and they do different exercises (squats, jumps, running,...). Points are recorded (number of squats, etc.). When the jumping is done, they move on to calculation tasks. Accuracy and speed of calculations generate final points. The team with the most points wins the title of the most sporty mathematician. Teams are named according to the most illustrative representatives of different seasons (flower, sun, pumpkin, snowflake). The types of exercises relate to corresponding animals.
ICT tools included	Drawing software (Paint) Scratch
Areas (select)	A1: Mathematics A2: Science A3: Physical education A4*: Informatics A5*: Art *Possibility of expanding activities
Topics (for each selected area)	A1: Multiplying a two-digit number by a single-digit number, dividing by 2 and 5, multiplying the sum, multiplying the difference
	A2: Change of seasons and duration
	A3: Aerobic and anaerobic exercises
	A4*: Introduction to the drawing program and Scratch
	A5*: Getting to know certain painting techniques
Expected prior knowledge of students	Knowledge of arithmetic operations with natural numbers up to 100 Knowing the seasons *Elementary knowledge of computer work
Expected outcomes	Determination of arithmetic operations with natural numbers up to 100. Understanding how the seasons change.
Expected duration of the activity	2 school hours, with the possibility of extending it to a sports and art day
Preparation of activities	Download a pdf/pptx of the introductory text Download the pdf of the graphics for activities 2 and 3 Download the pdf for activities 4-8 Download the pdf for activity 9 https://inamath.uniri.hr/jumping-math/ *https://scratch.mit.edu/help/videos/
A detailed description of all teaching activities	**Activity 1: An introductory text projected on a "smart" board.** *Vanya, a second-grader, infatuated, prepares books for math class, looks longingly out the window, and sighs.* *Maya, a temperamental teacher, flips through a math textbook and admires the beautiful sunny day.* *Maya (with a contented sigh):* Finally, the sun. Everything is green and birds are singing. It is so nice. Vanya, what's wrong with you? Are you again absent-minded…

Vanya: Everything is fine. The notebook is ready. But why mathematics when it's such a beautiful day? I know, we have to practice, but that multiplication and division are really boring.
It's summer outside, and we're looking at some numbers on paper...

Maya: Wait a minute, what do you mean summer? Do you know what season it is now? We talked yesterday in class... But you were all a bit tired... that spring fatigue started to bother the children too...

Maya puts the textbook down on the desk, clapped her hands, and cheerfully says: Everyone put your notebooks away. We're changing the plan. Today we're going to make that math of ours run and jump and have a lot of fun.
First, we must organize into groups. I want to see suns, pumpkins, snowflakes, and flowers.

Murmurs in the classroom. The children are a little confused, but they are already starting to argue about who is a pumpkin and who is a flower…

Maya: No, I'm going to tell you how to divide into groups. Does everyone know when this/her birthday is?

A chorus of "We know!" echoes throughout the classroom.

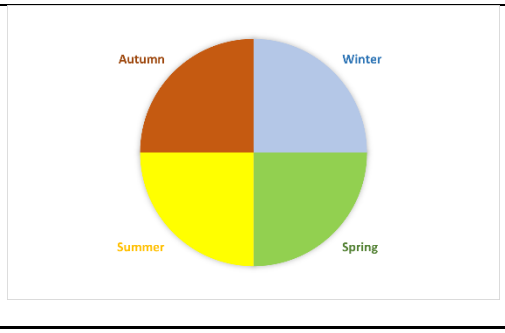
Maya: And do you know what seasons there are and when each season begins? We talked about it yesterday...

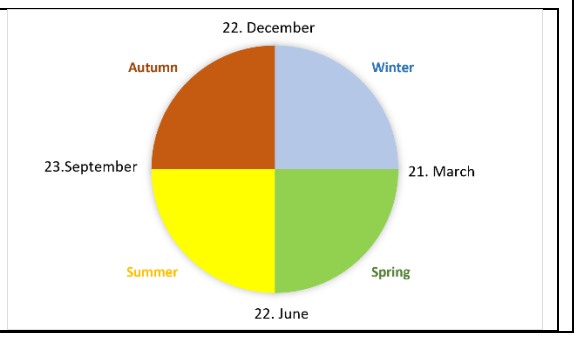
A less convincing "We know..." is coming.

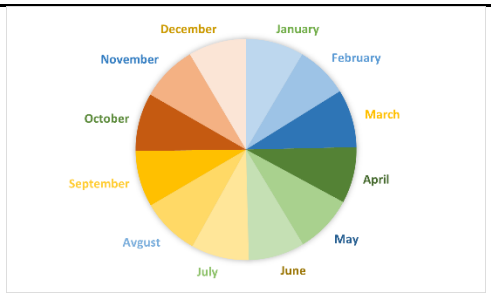
Vanya: Let me guess, I was wrong when I said it was summer.

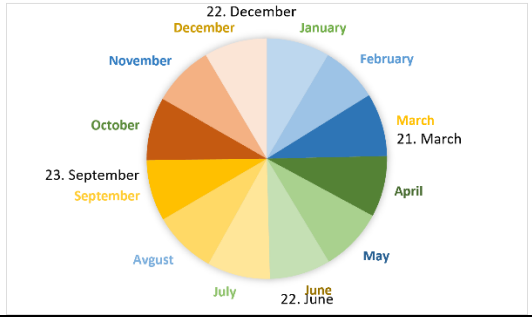
Maya: That's right. Don't worry, it's not terrible, now we will repeat and learn. Today is a beautiful spring day as if made for such a task.

Activity 2: Repetition of the seasons.
The teacher projects a graphic representation of seasons and calendar months on the blackboard. Children repeat the seasons and remember the corresponding dates that are entered in the drawing.



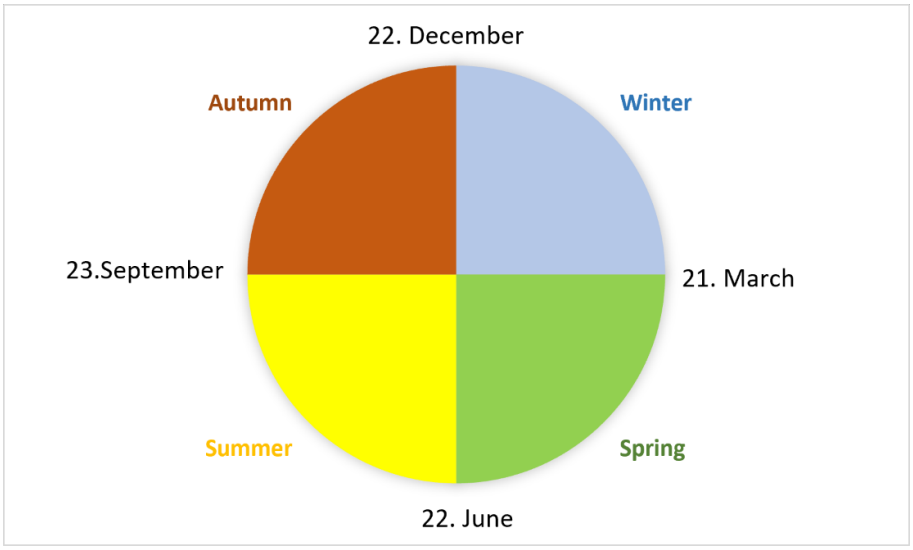






Maya: Let's organize you into teams.

Activity 3: Organizing the children into teams.
An image representing the seasons is projected on the "smart" board, with the exact dates when each season begins. Children approach the blackboard and, with the help of the teacher, are enrolled in the appropriate house according to their birthday.



Vanya: I got it! Snowflakes are those who were born in winter! Alas, I am a flower. My birthday is in April.

Maya: Well done! And now everyone to the yard! And take these tables. You will need to record the results.

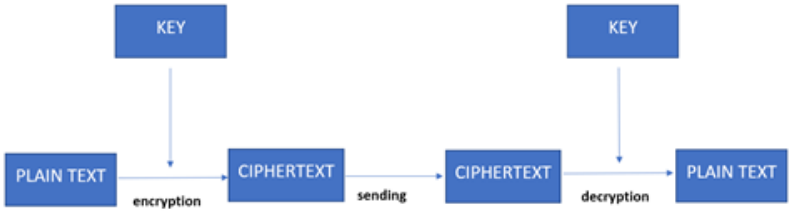
Activity 4: The frog number. Mark a 5m long track. Each child jumps in sequence across the marked distance and counts how many jumps it took. That number of jumps is the frog's number F.

Activity 5: The stork number. Balance exercise. The child stands on one leg. His/hers teammates make him/her laugh and count until he/she puts his foot down. The number they reached is the birth number S.

Activity 6: The gazelle number. Children ran across the yard with hops with high knee lifts. They count how many times they raised their knees. The knee lift number is the gazelle number G.

	Activity 7: The Meerkat number. Children count how many times they can squat and stand up without a break. The number of squats is the meerkat number M. **Activity 8: Teamwork.** Children fill in the team table. F in the team table is the highest F of all the children recorded in their tables. S in the team table is the highest S of all the children recorded in their tables. G in the team table is the highest G of all the children recorded in their tables. M in the team table is the highest M of all the children recorded in their tables. Together they count the following: • F*10; • S*3; • (G-F)*3; • (M+S)*4; • a two-digit number where F is the number of tens and the number of ones is 6, divide by 2; • a two-digit number where F is the number of tens and the number of ones is 0, divide by 5; • from a two-digit number where F is the number of tens and 8 is the number of ones, subtract 13 and divide the result by 5. The team that finishes first gets 30 points, the second 29, the third 28, and the fourth 27. Each correctly calculated result brings two more points, and for each calculation error, one point is lost. **Activity 9:** Teams should match the dates (December 22, March 21, June 22, September 23) with the corresponding names (winter solstice, vernal equinox, summer solstice, autumnal equinox) and descriptions (the longest nights, daytime and night last the same, the longest daytime). Each correct connection brings three more points to the team. The team with the most points wins and gets the title of the most sporty mathematicians, and in their honor, in the next art class, the seasons of the winning team are drawn.
Expanding activities	The possibility of organizing a themed day: **Activity 10:** Drawing the season of the winning team using different painting techniques. In agreement with informatics teachers, older students mentor younger students: **Activity 11:** Depending on the software available in the school, draw a team sign (pumpkin, snowflake, sun, flower) or animal from activities 4-7. **Activity 12**: Making simple scratch animations that illustrate activities 4-7. **Activity 13:** With the teacher's supervision, search the Internet to find interesting facts about the animals that appear in activities 4-7.
Additional notes	If a student attends classes according to an individual educational program, it is necessary to include a personal companion in the preparation and choose activities in which it is possible to include the student as much as possible.
Authors	Ivana Štajner-Papuga

Title	**The Secrets of Cryptography**
Key words	Encrypting and decrypting, Caesar cipher, dividing with remainder
Short description	Within this activity students are introduced to notions of encrypting and decrypting, using the secret key. The focus of the activity stands on algorithms of encrypting and decrypting with Caesar cipher. The algorithms can be developed by dividing the numbers to 100 with the remainder or through the use of adding and subtracting them to 100, depending on the students' prior knowledge. Tasks aim at the development of algorithmic and conducting procedures according to instructions. Students read a story and are introduced to historical development of Cryptography. In this way this activity correlates with Math, Science and mother tongue. Except for the development of Math skills, the aim of this activity is to show the implementation of Math in everyday life, especially its significant contribution in creating communication systems. After the main activity, the follow up tasks could be „Treasure hunt" and „What's the message?". This is to emphasize the difference between coding and cryptography. This activity can be further extended in IT classes and incorporated in English lessons.
IT tools	Micro:bit
Fields (select)	**A1: Math** **A2: Science** A3: Art A4: Music A5: P.E. **A6: Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Theme (for each selected field)	A1: division with remainder, adding and subtracting to 100 (exercise)
	A2: history of cryptography, timeline
	A6: reading comprehension
Expected prior knowledge	Adding and subtracting to 100, comparing numbers, dividing with remainder to 100 (activity can be carried out even if the students cannot divide with remainder)
Expected outcomes	• It uses natural numbers up to 10,000 to describe and display quantity and order. • Adds and subtracts in a set of natural numbers up to 1,000 • Multiplies and divides natural numbers up to 1,000 by single digits using long multiplication • It applies four computational operations and relationships between numbers in problem situations. • Solves tasks with one unknown by writing the letter as a number . • Uses different displays of data. • The student reads short texts thematically appropriate to the student's experience, language development and interests – finds data in the read text according to the instructions or questions • The student writes in the school handwritten letter letters, words and short sentences in accordance with language development.
Expected duration	45+90+45 minutes

of the activity	
Preparation of the activity	1. Teacher can deepen his knowledge of cryptography, if necessary. (https://web.math.pmf.unizg.hr/~duje/kript/osnovni.html) 2. Worksheet: The Codes Story (mathematical reading): https://inamath.uniri.hr/wp-content/uploads/2022/11/Prica-o-siframa.ej.docx 3. One can use Caesar disc: https://www.tinkercad.com/things/dyR9iVBxJ1W
Detailed description of all the activities	Students prepare individually before the activity (homework), worksheet: The Codes Story PART ONE: How did Caesar hide his messages? (45 minutes) Note: If students take IT classes, the first part can be done in their IT lessons and correlated with the topic of cryptography within IT curriculum. If students don't take IT classes, this activity could be covered in their Math or Science lessons. 1. Introduction: How do we encrypt and what is a key? (10 minutes)? With the help of pictures, students are introduced to the steps in encrypted communication. Encrypting is a process of substitution of plaintext with cyphertext, by using a certain key. In reverse, decrypting allows the person who knows the key, to read the encrypted message.  Having read The Story of Cryptography, we try to unveil what did Caesar use as a key and what is the key in scytale (with Caesar key is number 3 and with scytale key is the width of the stick). At this point of the activity, teacher can make and show a scytale. Even though, Caesar shifted letters for three places down the alphabet, today any shift of letters by a fixed number of positions is called Caesar cipher (k=1,…,25). 2. Encryption with Caesar cipher (10 minutes) Students are given the following task: Encrypt the word MATEMATIKA in Caesar cipher, with k=3. It would come in handy if we had Caesar disc or letters written in a circle. Students get the following ciphertext: PDWHPDWLND. We discuss the following questions: • How would you decrypt the given ciphertext? (We shift left for k=3 positions) • Is letter A always encrypted in D? (Yes) 3. Decryption in Caesar cipher (10 minutes) Students get the following task: Using the key k=5 we got the cyphertext HJEFW. What is the decrypted word? Students get the plaintext: CEZAR. • In this example we point that letters should be written in the form of a circle (a comment on encrypting letter Z) • In this example, we notice that 5 steps forward is the same as 26-5=21 steps back. 4. Caesar cipher in micro:bit (15 minutes) Students work in groups and each group is given micro:bits (one micro_bit for encryption and one for decryption).

Instructions to work with micro_bits:

- Key A: we choose the letter we wish to encrypt or decrypt
- Key B: we choose the key
- Keys A+B: we get a plaintext or cyphertext of the letters

The activity is carried out in the following way:
One student /group chooses a key and a short message (3-4 letters) which is encrypted with the key. Another student/ group decrypts the cyphertext using the same key.

5. EXTRA ACTIVITY (if there's time): How difficult it is to reveal the secret?

Students work in pairs (ex. two students at the same desk are a pair). Each student encrypts one word in Caesar cipher with the shift *k* (for the sake of simplicity and time saving we might set *k*<10). Each student defines the key *k* and encrypts a word, using this key (it can be a word in English, for example). He gives the cyphertext to his partner, without revealing the key. Partners now try to get the key used by their friends and reveal the original text by repeating the process of decrypting for each *k* between 2 and 9. Students will know that they've successfully acquired the key when they get a meaningful text.

<u>PART TWO: How do micro:bits (or computers) know what to do?</u> (90 minutes)

1. How do we persuade a computer to code in Caesar ciphers? (45 minutes)

Today, cryptography is done by the computers. However, computers can only perform the actions we order them, i.e. what we order in an algorithm. Luckily, computers are good in Math.

a. Introducing the process (15 minutes)

In this part we will introduce the process /algorithm for encrypting and decrypting in Caesar cipher.
We get students thinking about the process of encrypting by posing the following questions:

- Do we use Math in the process of encrypting with Caesar cipher? How would you explain the encrypting of a letter in Caesar cipher to the computer? Which operation would you use to write down the encryption: shift right in *k* positions (adding)
- Can we add letters and numbers? (no)
- Can we pair each letter with a number? (yes)
- Do we have a rule in putting letters to an order? (Alphabet, we pair each letter to their ordinal number in the alphabet).
- Imagine we encrypt with shift *k=3.* How would you move letter Z right in 3 positions? (We go back to the beginning: Z-A-B-C).
- How would you describe it in Math? (Calculating the remainders in dividing)
- What is the smallest remainder we can get? (Zero, that's why we'll mark letter A with number 0)

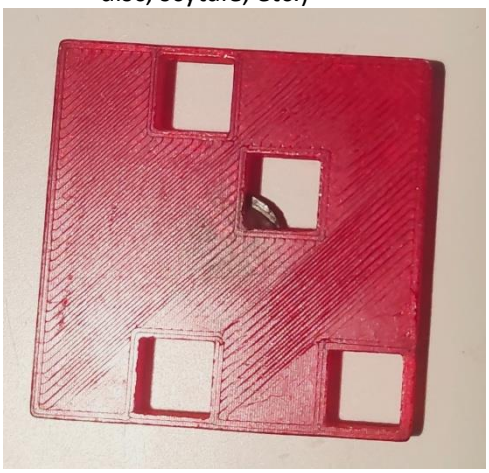
Then, we write down the algorithm for encryption in Caesar cipher.
Before the algorithm we choose the key *k* (a number between 1 and 25) and we pair the letters and numbers:

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

The process / algorithm:

i. we pair the letter we wish to encrypt with the number *x* (according to the table)
ii. we encrypt number *x* by calculating the remainder in dividing number *x*+*k* with 26

	or if *x*+k is less than 26, then we encrypt x by calculation *x*+*k* if *x*+k is bigger than 25, then we encrypt x by calculating *x*+*k*-26 iii. the result is a number between 0 and 25 and it defines the unique letter (from the table) We repeat the process for each letter of the alphabet. Students are not always familiar with variables x and *k* so we should explain how to insert values in the steps of the algorithm. b. Application of the process (30 minutes) Teacher assigns the following task: In applying the algorithm encrypt the word ZABAVA in Caesar cipher with key k=7. Students complete the process using the worksheet. We get the cyphertext of the word ZABAVA (key *k*=7): GHIHCH Students work together and identify the following: • Is letter A always encrypted in the same letter? (yes) • Do we need to repeat the process for each letter A appearing in the word (No, once is enough) • How many times do we carry out the encrypting process? (As many times as there are different letters in a word) 2. How do we "persuade" a computer to decrypt in Caesar cipher? (45 minutes) a. Introduction of the decryption process (15 minutes) We work together with students and come up with decryption algorithm. I. We pair the encrypted letter with number *y* (with the help of table) II. number y is decrypted by calculating the remainder in dividing number y+(26-*k*) with 26 or if *y*+(26-*k*) is less than 26, then we decrypt *y* to calculate *y*+(26-*k*) III. if *y*+(26-*k*) is bigger than 25, then we decrypt *y* to calculate y+(26-*k*)-26 and pair the acquired number with a letter from the table We repeat the procedure with every letter in the cyphertext. b. Application of the decrypting algorithm (30 minutes) The teacher assigns the next task, which is done on the worksheet: Using the algorithm to decrypt the text SZSNIJSVE encrypted in Caesar cipher with key *k*=4. Students reveal the message: OVOJEFORA HOMEWORK: Students are given homework task – they have to come up with their own encryption: they explain the key and describe the process of encrypting and decrypting. PART THREE (optional, in IT class): Implementation of the algorithm of Caesar cipher (45 minutes) • Students will create a program for Micro:bit which encrypti and decrypt in Caesar cipher (download the .hex file for micro:bit called *microbit-CEZAROVA-SIFRA* from https://inamath.uniri.hr/tajne-sifriranja-treasure-hunt/) • The selection of letters we want to encrypt and pair with their ordinal numbers (using lists). By pressing the key A, letters of English alphabet

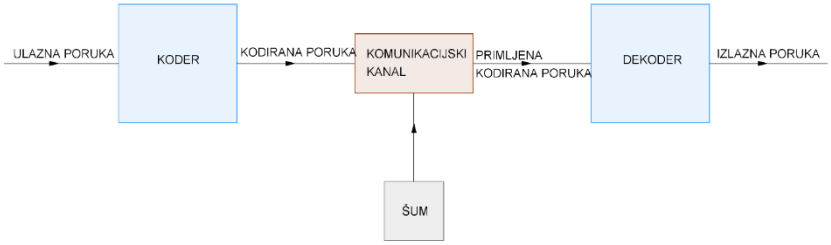
	are presented (in circle). • The choice of key (number) we wish to use in encrypting. By pressing the key B, numbers from 1 to 26 are shown. • The encryption process: pressing the keys A and B at the same time, the chosen letter encrypts according to the chosen key and we get the cyphertext.
Extension activities	1. Having completed the previous activities, „Treasure Hunt" game could be done in the following lesson. It should relate to encrypting and it should last for 45-60 minutes, according to the prepared scenario: https://inamath.uniri.hr/wp-content/uploads/2022/11/3_Tajne-sifriranja-Lov-na-Blago.ej.docx 2. To help students distinguish between coding and encrypting, we can teach the scenario: What's the message? 3. In extra Math classes, or with grades 5th – 8th, encrypting and decrypting algorithms can be introduced, as well as the implementation of algorithm in a program language (micro: bit, Scratch, Python). 4. In English classes Caesar disc can be used in teaching the English alphabet. 5. In Art classes students can make various „device" for encrypting (Caesar disc, scytale, etc.) 
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

Title	**What kind of message to send?**
Key words	Coding, Morse code, odd and even numbers
Description of the activities	In this activity, in a few simple examples (Chinese Whispers, The Morse alphabet) students are introduced to the theory of coding and its primary function: to detect and to correct the errors in communication. In two examples of communication between two computers (a code with repeating bits and a code with parity check), with the help of their teacher, students will analyse the number of errors occurred in the transmission of the information which are possible to detect and to correct. The aim of the activity is to develop algorithmic thinking and to show the implementation of Maths in everyday life. The activity shows the importance of Maths in creating the communication systems, and it stands as a great asset in the development of critical thinking in an entertaining way. Students read a story and they are introduced to historical development of the coding theory. This way this activity connects Maths and Science and their mother tongue. It is before or after the activity that teacher can do the activity called The Secrets of Criptography, to point the difference between coding and cryptography. This activity can be further developed in the IT class as well.
Digital tools	Micro:bit
Fields (select)	A1: **Maths** A2: **Science** A3: Art A4: Musi**c** A5: P.E. A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Theme (for each field)	A1: the importance of Math in everyday life and fostering a positive attitude towards Maths; odd and even numbers
	A2: Morse code, historical development of telegraph, timeline
	A6: Reading comprehension; language
Expected prior knowledge	
Expected learning outcomes	• Use natural numbers up to 10,000 to describe and display quantity and order. • Divides natural numbers up to 100 with the remainder • The student explains the organization of the time and displays the timeline of events • It places events associated with the immediate environment in the past, present and future. • The student listens/reads the media text formatted in accordance with the initial literacy and extracts important data.

	• The student creatively expresses himself according to his own interest motivated by different experiences and experiences of literary text
Expected duration	90 minutes
Preparation of the activity	1. If necessary, teacher can broaden their knowledge of coding: Basic terms in the coding theory (https://inamath.uniri.hr/wp-content/uploads/2023/02/linearni-kodovi.pdf) 2. Worksheet: The Story of Coding (Maths story): https://inamath.uniri.hr/wp-content/uploads/2022/11/The-codes-story.docx 3. Worksheet, Morse code: https://inamath.uniri.hr/wp-content/uploads/2022/11/Morse-code.docx 4. Prepare Micro:bit with the program for coding with repeating bits and Micro:bit with program for printing the coded message. From the site https://inamath.uniri.hr/kakvu-poruku-poslati/ you can download .hex files for Micro:bits. You need to download file *microbit-kod_s_pon_bitovima_KODIRA (*by pressing the key A you select the message you want to send (0 or 1) and by pressing the key B selected message is coded and sent to another Micro:bit) and file *microbit-kod_s_pon_bitovima_ISPIS* (by prrssing keys A and B at the same time MIcro:bit shows recived array).
Detailed description of activities	1. Students prepare themselves before the activity (homework), worksheet: The Story of Codes 2. Introduction (10 minutes) Game Chinese whispers: students play Chinese whispers to illustrate the fact that mistakes can occur in communication. 3. Motivation (5 minutes) How did a language originate? Do mistakes happen in communication? Can they be fixed by our brain? The students read a few sentences with mistakes: • Albert Einstein: “A mind is liek a parachute. It does ot work it if isnt open.” • Galileo Galilei: "We canot taech epople anithing; we can ony helpp them discover it witthin themeslves." • 0V0 J3 J3DN0574VN4 P0RUK4 K0J4 D0K4ZUJ3 D4 5M0 5P050BN1 R4ZUMJ37I 7EKS7 KOJ1 N1J3 N4P154N N4 574ND4RDN4N N4Č1N. Forty thousand years ago, people started developing language to communicate. Our languages function in such a way that our brains, the best decoders possible, manage to detect and correct numerous mistakes that occur in communication (ex. mispronouncing, lisping, misspelling...) We can point out that all the words in the Croatian language, around half a million, could be written in four letters or less. However, in this case, many words would differ in only one letter (ex. KIST i LIST) and many mistakes would occur in writing and they would be difficult to correct. It would be easier to notice and to correct the lapsus if the word would consist of ten letters or more. However, that would appear to be very inconvenient in writing and in speaking. 4. Morse code (20 minutes) Are there ways to communicate without hearing, at sea for example? In 1835 the Morse telegraph was created and it used Morse code.

Students are given the worksheet - Morse code. Each student comes up with a five – letter word and code it in Morse code. Teacher invites several students to share their message with other students with the help of a lamp.

5. How do the computers communicate? (10 minutes)
   Do you know how two computers or two mobile phones communicate? Do they communicate in Croatian? Or in English?
   Two computers communicate in the language of zeros and ones.
   Just as we make mistakes in speech and writing, the same things happen when two computers communicate. In the journey from one computer to another, a zero can be mistaken for a one, and vice versa. In speech or writing our brain converts a reasonable number of errors.
   Can computers correct the errors that occur in the transfer of information? It depends on us, i.e. on the language we prepare for the communication between the computers. Coding theory is a part of Maths, which uses Maths to invent different languages for computers to communicate. Today, we are going to introduce you to two languages for computer communication.



   Code is a set of words (a string of symbols) which a coder creates and sends to communication channel. Each word in a code is made of input message (information) that we want to send. Decoder receives a coded message and tries to establish the information, while trying to correct any possible errors in the communication channel.

6. Language for computer communication: Code with repeated bits (20 minutes)
   Let's assume we wish to transfer information written in binary representation – the length of 1. For example, the message is 1 and we wish to detect and correct the errors so we send the string in length 8: 11111111.
   Activity with Micro: bit
   Students work in pairs and each pair is given two Micro:bits. One Micro:bit is used to choose a message a student wishes to send (0 or 1), and the other Micro:bit is used for decoding (Micro:bit shows the message from communication channel).
   Students work together to come up with the conclusion on the possibilities of a code with repeated bits regarding detecting and correcting errors. They conclude that this code can detect the maximum of seven errors and it can correct the maximum of three errors.

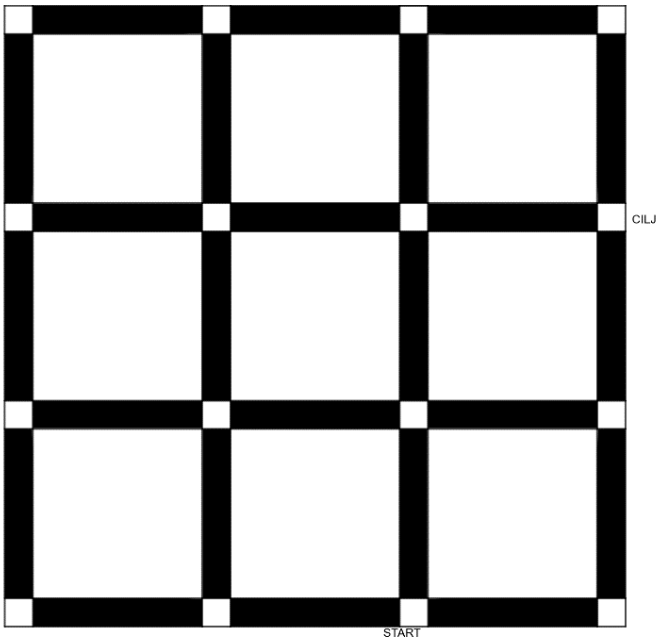
7. Language for computer communication: Code with parity check (20 minutes)
   Let's assume we wish to transfer information written in binary string, length seven, ex. 1110001. In transfer we add the eighth member of the string a: a = 0 if the starting string has even number of units, a = 1 if the starting string has odd number of units. In this example it

	means that we are sending the string: 11100010. Activity: Students work in threes: the first student is coder (He chooses the message, codes it and writes it on paper). The second student is communication channel (he receives the coded message and makes errors). The third student is decoder who tries to detect an error and to correct it. Students work together to come up with the characteristics of the code with parity check: we can detect the odd number of errors in transfer, but we can correct none of the mistakes. 8. Conclusion (5 minutes) In two examples binary strings length 8 were used in the transfer of information. In the first example, we transferred less information with more possibilities of detecting and correcting errors. In the second example we transferred more information with less possibilities to detect and correct errors. The main problem in coding theory is to find a code to transfer the right amount of information with appropriate possibility to detect and correct errors. Note: If the activity The Secrets of Cryptography was carried out, it is possible to point the difference between coding and cryptography in the end. The aim of coding is to correct the errors which occur in communication channel. The aim of cryptography is to "hide" the message, so that it can only be read and understood by those who are supposed to read it. Both processes are used daily, in communication via mobile phones, computers, watching TV.
Possible extensions of the activity	In IT class (or with 5^{th} to 8^{th} grade students) students program Micro:bit in several tasks: 1. Coding program for coding and sending message with a code with repeating bits. 2. A program for receiving, correcting and decoding code message with repeated bits. 3. A program for coding and sending message with code with parity check. 4. A program for receiving and decoding the code message with parity check and mistakes detection. These tasks can be carried out programming in other programming language (ex. Scratch or Python).
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

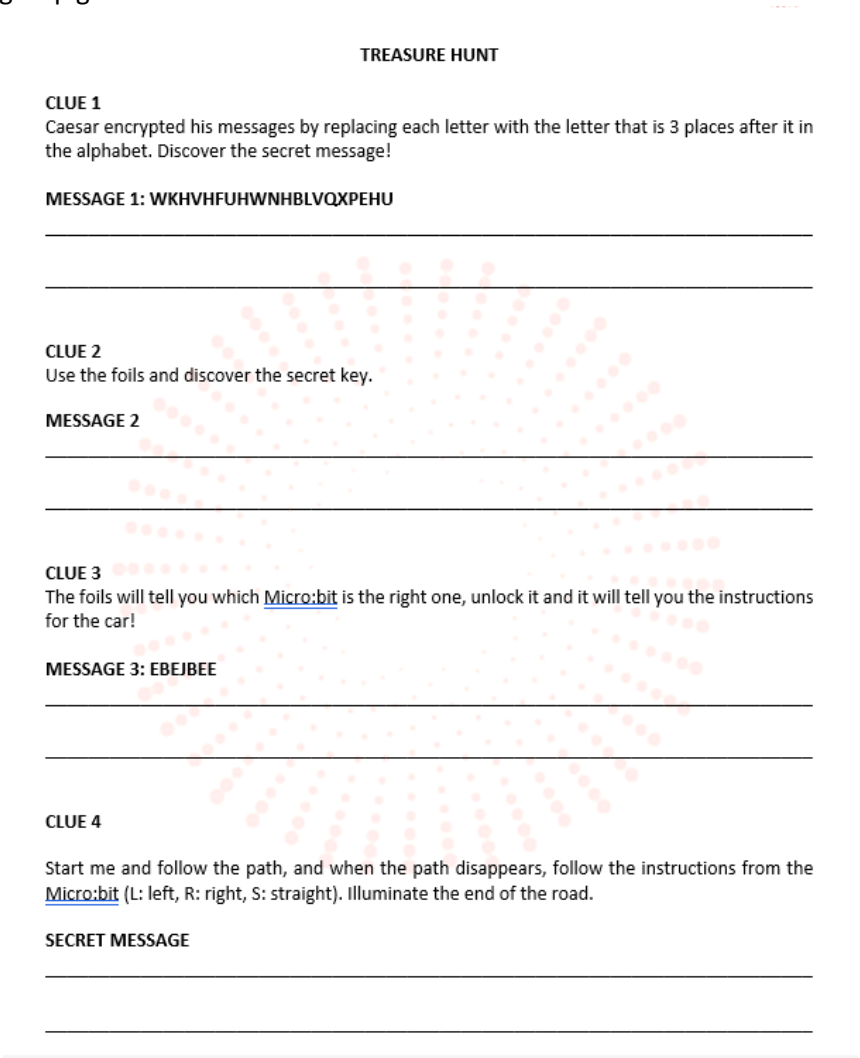
Title	**Time machine**
Key words	Timeline, reading comprehension, calculating to 2022
Short description	In this activity, mother tongue, math and science corelate through reading math stories. While reading stories, students deepen their literacy skills, along with their ability to find information in the text. In doing the tasks they need to look for the information in a text, to do calculations with the data they found and match the result with timeline. It is possible to adapt the activity to different topics, using the same concept, and to broaden it with activities in foreign language lessons, Art classes, IT classes
IT tools	3d print
Fields (select)	**A1: Math** **A2: Science** A3: Art A4: Music A5: P.E. **A6: Mother tongue** A7: Foreign language Other: A8: IT **A9: Civic education**
Themes (for each selected field)	A1: calculation to 2022
	A2: timeline, important people from the past, important people from their region
	A6: reading comprehension
Expected prior knowledge	Timeline, calculating to 2000; reading comprehension
Expected learning outcomes	• It uses natural numbers up to 10,000 to describe and display quantity and order. • Multiplies and divides natural numbers up to 1000 by single digits using long multiplication • The student shows the timeline of events and considers their importance. • The student navigates changes and relationships during time cycles and analyzes the connection of time cycles with events and important people in their homeland. • The student explains the results of his or her own research into nature, natural and/or social phenomena and/or different sources of information. • The student reads the text with understanding and finds important information in the text. • The student reads the text with understanding and recognizes the characteristics of the scientific text.
Expected duration	90 minutes
Preparation of the activities	1. Timeline (GeoGebra): https://www.geogebra.org/classic/dyregh3a 2. Timeline 3d print (https://www.tinkercad.com/things/6jV5SnU98GQ?sharecode=0i19ppAktZkphhaPLX0VQj4upB82syi3zE98xbhGWkg) A part of the timeline is prepared. There are 19 dashes on the timeline (the last dash is a joint of two strips) and it represents a period of 20 years. The same can be used in making the number line. If printed in length of 20 cm, the distance between the dashes is 1 cm. Each has a hanger, and it can be hung on the classroom wall and used as a hanger. 3. Slips of papers with questions, for students to write the year and hang it on

	the timeline. 4. Math reading: The Codes story (https://inamath.uniri.hr/wp-content/uploads/2022/11/The-codes-story-1.docx) The Story of Cryptography (https://inamath.uniri.hr/wp-content/uploads/2022/11/Prica-o-siframa.ej-1.docx) The story of the biggest Croatian mathematician (https://inamath.uniri.hr/wp-content/uploads/2022/11/Prica-o-najvecem-hrvatskom-matematicaru.ej.docx) The Numbers story (https://inamath.uniri.hr/wp-content/uploads/2022/11/Prica-o-brojkama.ej.docx) The story of one of the biggest Croatian discoveries (https://inamath.uniri.hr/wp-content/uploads/2022/11/Prica-o-jednom-od-najvecih-hrvatskih-otkrica.ej.docx) Topics in math reading can vary; significant people in a region, scientific discoveries and achievements, famous scientists...
Detailed description of the activities	1. INTRODUCTION (5 minutes) In introduction, we revise the terms: timeline, past, present, future, decade, century, millennium... 2. TIMELINE COMPOSITION (10 minutes) Students work together and compose the timeline. They mark the beginning of the century. They have to put more than 10 slips together, so the suggestion is that they work in four groups. Each group should compose a timeline for the period of 500 years. 3. MATH STORIES (45 minutes) The main part of the activity is done in groupwork (groups of 4 to 5 students), one story by one. Students read their story and answer the questions as a group. The first group to answer the questions, raise their hand and they are given a slip of paper with a question. They answer the question and hang their answer on the timeline. The group to hang their year correctly, get one point. Teacher and students give their comments on the story and answer the questions 4. THINK OF YOUR OWN TASK (30 minutes) In introduction, students do one task: I was born in January in 80th year of the 20th century. My sister is half my age today. What year was my sister born? In the end, students in groups have to think of their own task, following the example above. The answer should be a year. For a task well planned and solved correctly, a group gets one point. However, their tasks will be done in another group and those who are the fastest to do the task het a point. This is a demanding part of the activity. But, if they work regularly, and do various activities, if they create activities themselves, it should in time become less demanding, and their tasks will become more creative and complicated.
Options to extend the activity	1. Each student is assigned a century and has to choose one event or one person related to that century. They have to write a short text which should include a drawing done by students, inspired by the story they chose. If the timeline is hung on the wall, students can hang their drawings in the right places. 2. In IT lessons, students can design timelines (ex. In program GeoGebra or Tinkercad). 3. The activity can be incorporated in the foreign language classes. The text can be prepared in the foreign language.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

Title	**The Secrets of Cryptography – Treasure Hunt**
Key words	Encrypting and decrypting, Ceasar cipher, visual cryptography, Enigma, treasure hunt
Short description	Within this activity students are introduced to the theory of cryptography, as well as to some ways of encrypting used throughout the history: Caesar cipher, visual cryptography, public key cryptography. The activity is designed as Treasure Hunt game. Students work individually or in groups and they solve the riddles. Teacher helps if necessary. Students study the materials and get enough information to do all the tasks. Group work is encouraged, along with critical thinking and reading comprehension. There are two versions of the activity: a simple and a complex one. In the complex version students learn the ways of encryption with the help of Enigma. Activity can be further extended in Art classes (making of the visual criptography of an image) and in IT lessons.
IT tools	Micro:bit, Maqueen, 3d print
Fields (select)	**A1: Math** A2: Science A3: Art A4: Music A5: P.E. A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: dividing with remainder, prime numbers
Expected prior knowledge	
Expected learning outcomes	• Uses natural numbers up to 10,000 to describe and display quantity and order. • Adds and subtracts in a set of natural numbers up to 1,000 • Multiplies and divides natural numbers up to 1,000 by single digits using long multiplication • Applies four computational operations and relationships between numbers in problem situations. • Solves tasks with one unknown member by writing the letter as a number
Expected duration	45-60 minutes
Preparation of activities	1. Teacher can deepen his knowledge of cryptography, if necessary. (https://web.math.pmf.unizg.hr/~duje/kript/osnovni.html) Preparation of the material is necessary. Materials are available at: https://inamath.uniri.hr/treasure-hunt/ 2. The riddles (tasks) booklet for the Treasure hunt includes decrypting tasks, using various methods of decryption (Caesar cipher, visual cryptography – foil, public key cryptography at micro:bit) 3. Preparation of the foil set for visual cryptography (two foils overlap and thus reveal a picture): foils should be printed, using

	transparency sheets for the overhead projector. 4. 5 micro:bits should be prepared in the following way: four „fake" micro:bits are necessary (with prepared code Microbit_false.hex) and four labels should be attached to them (5 petals flower, empty circle, five-pointed star, a sad smiley face) and one „real" micro:bit (with prepared code Microbit_real.hex) with the label with a black square. 5. Preparation of the Treasure hunt path to end the activity. Those are the parts of the road and a plan of the neighborhood that will be composed of pieces of paper or made in 3D printer (https://www.tinkercad.com/things/8ayryvLRFLL), I.e. a neighborhood from the scenario Math Ride (on the link https://inamath.uniri.hr/math-ride/ you need to download GeoGebra files: voznja, voznjaT, voznja kut i voznja ravno; and print them in order to get parts of the road). A small piece of paper (or an envelope) should be placed on each crossroads and something should be written on it (using the invisible UV marker) A message (ex. Good job!) should be written in the field CILJ (The end).  6. A car toy for kids to steer (manually, according to instructions) or to program Maqueen according to the prepared codes: *microbit-DaljinskiZaMacqeen6* and *microbit-MacqeenPlusCTKPapir23* from the page https://inamath.uniri.hr/math-ride/ 7. Instructions and description: the process of cryptography, Caesar cipher, Enigma, cryptography with the help of Micro:bit 8. Tasks booklet for the advanced version 9. Preparation of Enigmas (for advanced version): https://www.thingiverse.com/thing:5793033
Detailed description of activities	1. This activity can be done after the scenario The Secrets of Cryptography, but it can also be done independently for students to learn about cryptography through a game. 2. Treasure hunt – a basic version (45 min) Students work in four groups. In this case, four sets of foils and four sets of Micro:bits should be prepared and placed on four separate tables. Each

group gets one booklet with riddles:



TREASURE HUNT

CLUE 1
Caesar encrypted his messages by replacing each letter with the letter that is 3 places after it in the alphabet. Discover the secret message!

MESSAGE 1: WKHVHFUHWNHBLVQXPEHU

_____

_____

CLUE 2
Use the foils and discover the secret key.

MESSAGE 2

_____

_____

CLUE 3
The foils will tell you which Micro:bit is the right one, unlock it and it will tell you the instructions for the car!

MESSAGE 3: EBEJBEE

_____

_____

CLUE 4

Start me and follow the path, and when the path disappears, follow the instructions from the Micro:bit (L: left, R: right, S: straight). Illuminate the end of the road.

SECRET MESSAGE

_____

_____

Having completed the 3rd clue with the help of Micro:bit, a group gets to the table with the path and the car (or Maqueen). Following the instructions they found, they should reach the finish field that has a message (Good job!, for example). If they made a mistake, the students go back to their seats and try to find the mistake and/or start again.

3. Treasure hunt – advanced version (60 minutes)

CLUE 1
Caesar encrypted an alphabet letter by replacing it with the letter 3 positions down in the alphabet. Guess a number less than 10 and discover a secret message: YTRNRFLNSJIYMWJJSZRGJWXQCCQQC

CLUE 2
Listen carefully to what Caesar says, lock the enigma, it will tell you what to do next.
OVDIDZPYOSDCYIUHTBDK

CLUE 3
Put the transparencies together to figure out the secret key.

CLUE 4

	Transparencies help you choose the right Micro:bit. Unlock it and discover the robot's path to the treasure: EBEJBEE. CLUE 5 Run me and follow the path, and when the path disappears, listen to what the Micro:bit tells you (L: left, R: right, S: straight).
Extension activities	1. Students get to know the procedure of making visual cryptography foils (https://www.101computing.net/visual-cryptography/) and they can make visual cryptography foils in Art class or within lesson on the geometrical shapes. 2. In IT classes students can change and/or make an encrypting / decrypting program in Micro:bit and change / make the program for micro:Maqueen. They can also make a path in the program GeoGebra (paper path) or in Tinkercad (3d printed path).
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

Title	**Math trick with cards: Squared trick**
Key words	Math trick with cards, calculation up to 100, two-dimensional table
Short description	This activity is based on Math card tricks, and it develops a positive attitude toward Math. This activity is a great way for strengthening early and basic Math skills in a fun and creative way which increases their interest and activates their desire to succeed in class. The Math basics for this trick (Squared trick) include calculations up to 100, division with remainders, and working with a two-dimensional table in which the position is defined by ordinal numbers of columns and rows. This activity develops students' motors skills, their abilities to observe and reproduce the procedure and their ability to focus and carry out the procedure according to instructions. In this activity success depends on the successful completion of each step. It is possible to extend the activity through creative performance withing the mother tongue classes, as well as Art and Music classes.
IT tools	
Fields (select)	A1: **Math** A2: Science A3: Art A4: Music A5: **P.E.** A6: **Mother tongue** A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each field)	A1: early mathematical skills, basic mathematical skills: repetition of the process in fixed order; calculating to 100, division with the remainder
	A5: motor skills and fine motor coordination skills in working with hands
	A6: creating stories; communication with audience, giving clear instructions
Expected prior knowledge	Calculating up to 100; division with remainder
Expected learning outcomes	• It uses natural numbers up to 10,000 to describe and display quantity and order. • Adds and subtracts in a set of natural numbers up to 1000 • Divides natural numbers up to 100 with the remainder. • He actively participates in elementary games that develop self-esteem, self-confidence and perseverance. • The student reads/listens to short texts thematically appropriate to the student's experience, language development and interests • The student talks and speaks in accordance with the theme of everyday life and respects the rules of polite conduct. • The student finds the data in the read text according to instructions or questions.

Expected duration	90 + 45 minutes
Preparation of the activity	1. Prepare card decks (one deck per two students). Twenty-five cards per each student suffice, so one deck can be used for four students. 2. Teacher practices the trick. 3. Teacher acquires math calculations in the base of the trick, according to instructions. (https://inamath.uniri.hr/wp-content/uploads/2022/11/Trik-na-kvadrat-upute-za-provedbu-trika.ej.docx) 4. Worksheet: Squared trick lesson plan (https://inamath.uniri.hr/wp-content/uploads/2022/11/Trik-na-kvadrat-priprema-radni-listic.ej..docx) 5. Worksheet: Squared trick generalization (https://inamath.uniri.hr/wp-content/uploads/2022/11/Trik-na-kvadrat-poopcenje-radni-listic.ej.docx)
Detailed description of the activities in class	**1.** PART: Preparation of the activity (15 minutes). The activity starts by doing the worksheet which can also be assigned as homework, before the lesson. After the activity it is important for the teacher to comment on the acquired results and for all the students to recognize that numbers of rows and columns are exchanged in tasks one and two. **2.** PART: Teacher presents the trick (20 minutes) Teacher announces that he is going to show a math trick with cards. To do the trick it isn't necessary to have skilled hands, but to perform the announced procedure and to let Math do the magic part. Teacher starts the presentation by explaining what a math card trick is; he / she explains that Math tricks rely exclusively on Math, and that there are neither special skills required, no „fake" and marked cards, nor hidden information. In the introductory part it is pointed out that the most important part of the activities is: • Students listen carefully and follow the instructions. • Students perform what teacher instructs them, in a careful, slow and concentrated way. • Students don't play with cards, after each step they stop and wait for the further instructions. To perform this trick, the teacher needs to take 25 cards from each deck. If there shouldn't be enough cards, it can be carried out with p^2 cards, so 16 cards should be acceptable (if the trick is done with p^2 cards, number 5 from the trick instructions should be substituted by p). Teacher does the trick several time, according to the instructions, without explanations. Students observe. It is important that both teacher and students are positioned in such a way that everyone can see the trick. This is easily accomplished if teacher takes a seat on the floor and students gather around him / her. This part of the activity usually leads to excitement, disbelief and rounds of questions like *How did you do that?* The teacher asks the students if they would like to know how and why does this trick works and if they would like to learn how to do the trick.

3. PART: Disclosure and explanation of the trick (30 minutes)

In this step the teacher explains why does this trick work, he elaborates the math behind the trick. This is the key part in math tricks because the aim is for students to understand why does the trick work, because that is the only way they will understand that the trick is not based on scams but on math science and that there is a logic explanation why the trick works.

1. The performer (teacher) asks a helper (a student) to mix the cards well. He deals the cards in 5 decks of five cards, facing head up. While he/ she is dealing the cards, he/she asks his/ her helper to pick one card and to remember the deck of the card. We mark that deck as S (this is announced to all the students, so they'd be able to follow the card). Compared to the table from the preparatory activity, the students decide on the column. The teacher shows the table from the preparatory activity and marks the chosen column in a different color.

S0	S1	S2	S3	S4
0	1	2	3	4
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19
20	21	22	23	24

We should notice that in each column there are cards (marked with numbers 0 – 24) that, when divided by 5, give the same remainder, in each column these are: 0, 1, 2, 3, 4.

2. Teacher takes the piles of cards from left to right and the first pile should be on top facing head down and the last pile should be on the bottom, face down. He asks students about the order of the cards in the deck.: Are the cards piled in the same order as in the beginning? What changed?
By observing the table, they conclude that the order of the cards changed, and now the cards from the first column are on top of the deck. The order is:
0,5,10,15,20,1,6,11,16,21,2,7,12,17,22,3,8,13,18,23,4,9,14,19,24

3. Teacher deals the cards face up in five piles so that each card falls in the following pile (as he did in the previous dealing). While dealing the cards he asks his helper to memorize the pile in which he saw the card and to share that with the other students.
Teacher asks students which cards are now in the first row. They conclude that now in the first row there are cards that were in the first column in the previous activity. The order of the cards is shown in the table.

R0	R1	R2	R3	R4
0	5	10	15	20
1	6	11	16	21
2	7	12	17	22
3	8	13	18	23
4	9	14	19	24

They can conclude that the cards from one column are in one row and that they have the same number (number S). (Students are not surprised thanks to the preparatory activity).

The teacher asks his helper to show the column in which he / she saw his / her card. We mark the number of the pile with letter R and color it with special color in the table from the beginning of the activity.
Now the teacher knows that the card was in column S and in row R.

4. Teacher deals the cards again, but now the cards are shown and placed in the shape of a square. First, he deals the five cards form the top row, and then all the other rows. Students notice that now the cards will be laid in the same order as in the first dealing and that the card they are tracking will be in column S and row R.

	S0	S1	S2	S3	S4
R0	0	1	2	3	4
R1	5	6	7	8	9
R2	10	11	12	13	14
R3	15	16	17	18	19
R4	20	21	22	23	24

The notice that the card (before the last deal of the cards) in the deck was R*5+S+1 in order (if we start counting from 1) or R*5+S in order (if we start counting from 0).

4. PART: Practice of the trick (25 minutes)

Now, we switch the activity to students and they try to do the trick, by following the procedure the teacher showed. Students work in pairs, they do the trick as many times as it takes them to practice it, and to perform it flawlessly.
It is important that all the pairs do the trick simultaneously (all pairs for the same number), following the instructions from the teacher. It is also important that students control the performing of the trick.
Dividing the number with the remainder can be done on paper.

Their homework task is to practice the trick and to show it to their parents.

5. EXTRA ACTIVITY: GENERALIZATION OF THE TRICK (45 minutes)

- Teacher's presentation of the second part of the trick (10 minutes)

Having acquired the basic trick, it can easily be generalized. Before the start of the trick, ask your helper to pick a number from 1 to 25.
The trick is carried out in almost identical way (according to the instructions), but in this extension, we place the card on the place that the helper chose in the beginning. We can do that if we pick the piles of cards carefully.

In the end of the trick, we can ask the helper to search for his card by himself, and it should be B cards away from the beginning of the deck.

- Disclosure and explanation of the second part of the trick. (20 minutes)

Before they start the trick, the teacher asks his helper to pick a number

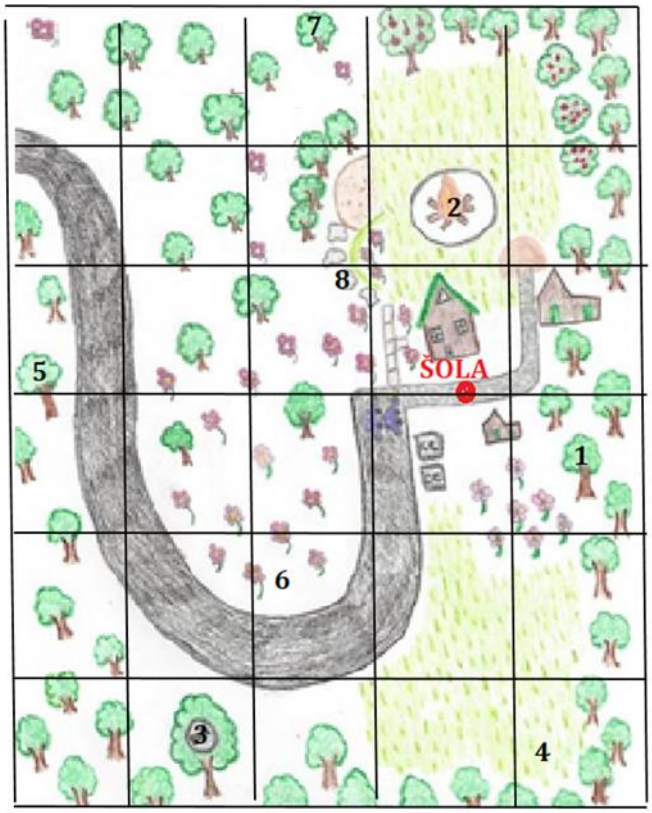
	from 0 to 24. We will mark this number as B (for example let's say that B=11). We divide number B with 5. Result is number R and remainder is S. In our example R=2, S=1. This part can be shown on the blackboard to show it clearly. After the first deal the cards are arranged as shown in the table. We notice that numbers in columns all give the same remainder when they are divided by 5 and that this remainder equals the number that marks the column. We also see that all the numbers in rows give the same result when divided by 5 and that are equal to the number that marks the row. If we calculate numbers of rows and columns beforehand (numbers R i S), we can place the chosen card on that spot such that we decide carefully where to put the pile, in the following way. • After the first deal, the performer collects the piles but watches for the chosen pile to be placed on the place S, which we calculated earlier. • After the second deal, the performer collects the piles but is careful to put the chosen pile to place R. Now the performer has the deck in which the chosen card in in place B=R*5+S (if we start counting from 0). • Practice of the trick (15 minutes) Now students perform the activity, doing the procedure that the teacher showed. They work in pairs and practice, until they perform it impeccably. Dividing the number with the remainder can be done on paper. For their homework students practice the trick and present it to their parents (students are given written instructions). Students are motivated to do the trick without paper, and to calculate by heart.
Extension options	1. Creating a story and adding details which will make the trick interesting for the audience (ex. Coming up with questions and procedures to make the final guessing more dramatic) 2. Creating or finding the music and scenery for the performance. 3. Practicing and performing of the trick. Every trick is a little show that performers have to practice and perform in front of the audience. Trick demands communication with audience, students give clear instructions and guide the audience through the story they invented, to make the trick more than a math operation.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

	S0	S1	S2	S3	S4
R0	0	1	2	3	4
R1	5	6	7	8	9
R2	10	11	12	13	14
R3	15	16	17	18	19
R4	20	21	22	23	24

Title	**Orientation**
Key words	grid, sketches, maps, orientation games
Short description	Orientations are given in various subjects from pre-school education onwards. Better orientation in cartographic lessons can be achieved by adapting basic cartographic elements and introducing them in the early grades. Once students have mastered the basic concepts of orientation in real space and on a piece of paper, we build on this knowledge using a map. With the cross-curricular unit Orientation, students learn the strategy of reading and navigating on the grid.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: **Mother tongue** A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: the strategy of reading the grid and orientation in the grid.
	A2: orientation on sketches and maps
	A5: playing a team orienteering game on a marked route
	A6: precise expression
	A7: basic terms for orientation
Expected prior knowledge	basic orientation in grid; understand the terms left, right, up, down, forward, backward; sketches, maps
Expected learning outcomes	LEARNING OBJECTIVES: MATHEMATICS - to learn about the strategy of reading the grid and orientation in the grid. NATURAL SCIENCES - to know how to use different types of sketches and maps. ENGLISH - to recognise and understand the terms left, right, up, down, forward, backward, - to follow instructions in English by responding to them with movement. PHYSICAL EDUCATION - to develop navigational skills by playing a team orienteering game on a marked route around the school. - to improve general aerobic endurance by playing an orienteering game along a marked route. SLOVENE (mother tongue) - to develop orientation on the body, in space and on paper, - to observe objects in a picture, ask about their position or movement using the correct interrogative and express their position

	using the correct proposition.
Expected duration	2 x 90 minutes
Preparation	TEACHING RESOURCES AND MATERIALS: Appendix 1 : GRID WITH SHAPES Appendix 2: ZOO MAP
Detailed description of activities	**1. INTRODUCTORY PART / WARM-UP** The game "Simon says" (in English) We revise the terms left/right, up/down, forward/backward with the pupils with the game 'Simon says'. Examples of instructions in English: Raise your left/right hand. Look up/down. Jump forward/backward. Do a double-leg jump forward/backward. Make two/four/seven steps to the right/left. Touch your schoolmate's left leg with your right hand. Students are given a grid with numbers from 1 to 25 (5x5). The game is played in pairs. They take turns to choose any number and try to guide their classmate to their number by giving instructions. They can start at 1. Eg: Go one step right. Go two steps up. Go three steps right. Go one step down. - Is it number 10? - Yes. We continue by learning about the grid in English (see also "Working with the grid" in the main part). Prepare a "Treasure map" (as in the appendix; choose the appropriate map elements to use the level-appropriate vocabulary; e.g.: treasure, forest, crocodile, trap, mountain...), where students answer the questions. Finally, the pupils work in pairs to make a "treasure map" by themselves – we give them a blank grid with a map and pictures to stick on the grid. They can then play the "treasure hunt": For example, a pupil with a map says: Start at B,5. The treasure hunter has a blank map and needs to guess. He/She can only move one space at a time. E.g. Go to B, 4. Go to C, 4. On their journey, may come across a trap or e.g. a crocodile and have to go back to the beginning. Once the first pupil has found the treasure, the pupils can switch roles so that the second pupil can also try their search skills. **2. MAIN PART** Working with the grid We show the grid on the transparency. Discuss the grid with the pupils (where the columns are, where the rows are, how the columns are labelled and how the rows are labelled). Together, we determine the coordinates of the first shape and write them on the board. The pupils are given a grid worksheet (Appendix 1), in which they first write letters and numbers and then draw the shapes with the help of a template as instructed. At the same time, the activity is also carried out on the board (whiteboard). Examples of instructions: - Draw a red circle in the box (A, 1). - Draw a green rectangle in the box (A, 4). - Which box is the black rectangle located in? Draw it. - Draw a purple triangle in the box (A, 3), - The next two drawing instructions are given by the learners.

	The ZOO grid. Learners answer the questions on the worksheet (Appendix 2) using the ZOO grid. **Making a map** In pairs, the pupils assemble the map into a grid using the coordinates written on each piece of the map (Appendix 3). They first place the pieces of the map in the grid but glue them together only after the teacher confirms that the map is correctly assembled (we must make sure that each piece is oriented correctly). **Map** Together we look at the map (a simple map of Slovenia, with pictures showing tourist activities) that they made in the previous activity and talk about it with the pupils: - What did you put together in the grid? Why do we need maps? Have you ever seen a map like this? - Name some of the places that are shown on the map. Have you been to any of these places? What was the purpose of your visit/Why did you go there? - Let's find a river on the map. What colour is the river? - What do you think the pictures on the map show us? We cannot know what all the pictures mean, so we need a legend. We hand out the map legend and we look at it together. We discuss what we would call such a map. - Then we carry out an orientation activity on the map using a grid e.g. What can we do when we are on holiday in (B, 4)? Which places are in this box? Is there a river? etc. We also look at other types of maps (map of your hometown, car map, mountain map, interactive maps, etc.). **3. CONCLUSION** Orienteering game (the number of stations can be adjusted according to the time available or the school's surroundings) To prepare, you will need: skipping rope, balls, cones, balls, or vortexes, as well as envelopes with the tasks, cards, and pens for each group.



Instructions for pupils: Using a map showing the surroundings of our school, orient yourself and find the 8 envelopes located at the marked points (marked with numbers 1 to 8). In each envelope, you will find a task to solve. Each group will receive a pen and a card at the start of the activity on which they will write down all the solutions. The first group to get to the finish line first and to also solve all the problems correctly wins.
Start: run from the school to the tree marked with the number 1. Near the tree you will find an envelope with the 1st task waiting for you.
1: On all fours crawl around the tree. When you have done this, look at the tree and write the name of the tree on your card. Then find number 2 on the map, run to it and look for envelope number 2.
2: Perform frog leaps around the fireplace. Then write down on your card who to call (and what number) when there is a fire. When you have completed the tasks, find number 3 on the map, run to it and look for envelope number 3.
3: (Preparation instructions: the teacher places the balls into a ring right next to the tree. Then place a cone ten metres away from the tree.)
There are balls waiting for you under the tree. Each person takes a ball and takes turns leading it to the cone and back. Put the balls back where you got them. Then look at the tree and write on your card whether it is a conifer or a deciduous tree. Explain/give a reason for your answer. When you have completed the tasks, find number 4 on the map, run to it and look for envelope number 4.
4: (Preparation instructions: near the point number 4, the teacher prepares balls in a ring and sets up 6 cones in a slalom pattern.)
Each person takes one ball from the ring and guides it with their feet around the cones and back. Then write down on the card at least three grassland plants you see nearby. When you have completed the tasks, find the number 5 on the map, run to it and look for envelope number 5.
5: (Preparation instructions: The teacher prepares the skipping ropes and

	places them in the ring.) Each person takes one of the skipping ropes and does 20 jumps in one go. When you have completed the task, look at the leaves of the tree. Write down what the leaf edge looks like (serrated, wavy or smooth). After completing the tasks, find number 6 on the map, run to it and look for envelope number 6. 6: (Preparation instructions: Teacher prepares balls or vortex balls in a basket.) Each person takes one ball/vortex and throws it from the spot for distance. Repeat the exercise 3 times, then return the balls/vortexes to their place. When you have completed the task, write down on your card in which garbage bin you should throw away the juice container from which you have just drunk the juice. After completing the tasks, locate number 7 on the map, run to it and look for envelope number 7. 7: Hop around the two nearest trees and repeat 3 times. After completing the movement task, make an imprint of the tree trunk on the card. When you have completed the task, find number 8 on the map, run to it and look for envelope number 8. 8: Good job! You oriented yourself well on the map and completed all the tasks. Prepare the card with your answers and show it to the teacher to check if you have successfully solved all the tasks.
Extension activities	Using prepared materials, similar activities can be carried out in higher grades with more complex orientation maps and sketches and more complex tasks.
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanič, Mojca Žefran, Tadeja Volmut (Department of Elementary school, University of Primorska)

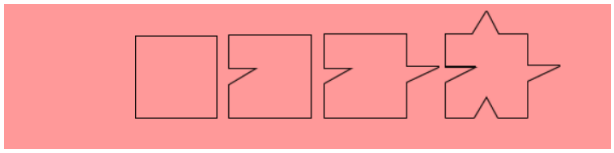
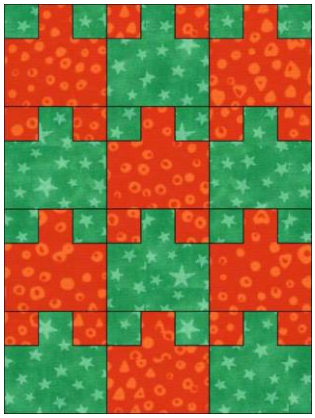
Title	**Time**
Key words	calendar, months, the days of the week, hour, minute, second, compare data
Short description	Unlike length, volume, and mass, time cannot be seen or felt, so measuring and understanding time is much more abstract for students than the other quantities we cover at the beginning of school. In this unit, students will combine their knowledge of math, science, language, and physical education to develop a holistic understanding of time and get a sense of how long it takes. It is important that students develop a sense of the duration of hours, minutes, and seconds and realize that a given unit of time lasts the same amount of time no matter what they are doing.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: **Mother tongue** A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: measuring time (minute, second)
	A2: calendar: the months, the days of the week
	A5: detecting and monitoring speed and endurance (60, 300 and 600 m)
	A6: TV schedule
	A7: the months and days of the week in English
Expected prior knowledge	Counting to 1000; calculating to 1000; know the time course of events, use some basic terms to define events (before, then, yesterday, today, tomorrow, week, days of the week, day, month, seasons, year); short distance running from a standing start; the days of the week in English
Expected learning outcomes	MATHEMATICS - to know and choose (according to the situation) the appropriate units of measurement for measuring time, - to estimate, compare, measure quantities, and record the measurement using numbers and units of measurement, - to calculate with one-name units of measurement, - to present data in a table and in bar charts - to read the table and the bar charts. SLOVENE (mother tongue) - to find relevant information in the TV guide. NATURAL SCIENCE - to know how to divide a day into hours, hours into minutes; know how to measure short-term events, - to learn that time management (timekeeping) is essential for successful learning. ENGLISH - to revise the months and days of the week in English using the calendar - when revising the days of the week in English, they use the

	structures "yesterday was ...", "today is ...", "tomorrow is ..." - to understand and use the prepositions before/after PHYSICAL EDUCATION - to use and understand different units of measurement (minute, second), - to learn how to measure/time various types of running (60m, 300m and 600m), - to present and compare data (personal achievements) using graphical displays.
Expected duration	2 x 90 minutes
Preparation	TEACHING RESOURCES AND MATERIALS: Appendix 1: A3 calendar Appendix 2 – Calendar worksheet Appendix 3: BINGO Appendix 4: TV guide worksheet Appendix 5: personal sports chart
Detailed description of activities	1. MOTIVATION: Riddles to introduce today's topic: USE RIDDLES IN YOUR MOTHER TONGUE Examples in Slovene: Teče, teče, nima nog, kaže, kaže, nima rok, čas računa brez glave, kar želiš brez ust pove (ura). Meri čas in tik taka, piska al' zvoni, dokler ne zbudi junaka, ki brezskrbno spi (F. Ankerst - budilka). Kaj nastopi vsako zimo, ko december se izteče in si vsi ljudje želimo zdravja in miru ter sreče (A. Štefan – novo leto). Students guess that today's activities will be time related. As the New Year has just started, we will look at the calendar for 2022. Calendar orientation: Each pair of learners is given an A3-sized 2022 calendar (Appendix 1) to answer the questions on the worksheet (Appendix 2 - Calendar worksheet). When they have finished, we check the answers. What do the numbers in each month under the letter T (to the left of Monday) stand for? – It is the number of the week in this year. How many weeks does a year have? All the learners look at the calendar and search for 5 January 2022 - the date when Tine was born. - When will Tine be 3 months old? - How old will Tine be on 5 August? - When will Tine be 1 year old? - When will Tine turn 6? - How old (in days) is Tine today?

	2. MAIN PART The duration of a second and a minute and the relationship between them - developing a sense of the duration of time - Learners take a piece of paper and estimate how many four-letter words they will write in one minute. Each learner writes his/her estimate with a number. On cue, they start writing the words. They write the words until the teacher gives the sign for the end, even if they exceed the predicted number of words. After exactly one minute they finish. Learners count the words and say the difference between their estimate and the actual number of words. - Learners stand up, put their chairs at the desk and start doing squats on cue, counting how many squats they have done. After one minute the teacher stops them. They write down what they think, how long they have been doing squats and how many squats they have done. They report the numbers to the class. - Learners sit on their chairs, close their eyes and have to be completely silent for one minute. After one minute, they write down what they think about how long they were silent. Then we talk about what they thought took the longest - writing the words, squatting or silence. What did they find the shortest? We tell them that each activity lasted exactly one minute, but that the duration of a minute can be experienced differently depending on the activity. What goes by faster - playing a computer game or writing homework? We link the conversation to the subjective experience of time. These activities can be done in English: we revise how many seconds there are in one minute. Count the 60-second period together with a stopwatch to get a sense of the duration of one minute (https://www.youtube.com/watch?v=U03lLvhBzOw). Learners say how long it takes (in minutes or seconds): one school lesson, brushing teeth, a football match, writing homework, one squat, etc. Revision of telling the time - Learners take a quiz to revise telling the time on an analogue clock. BINGO game. Each learner is given a bingo card with four clocks showing the time (Appendix 3). Each learner looks at the card and then "reads" the time shown on their card to their partner (neighbour). Then the game begins. The teacher takes the slip of paper with the time written on it out of the bag and reads it aloud, the pupils who have the clock showing the time cross it out. The first pupil to cross out all four clocks wins. The teacher prepares the slips of paper with the following times: 4.45, 4.15, 9.00, 6.30, 7.00, 8.45, 1.30, 8.00, 11.30, 9.15. 2. CONCLUSION Revision using a TV guide - analogue time display. We look at the TV guide (Appendix 4) with the learners, comment on it, then each learner chooses what he/she would like to watch that day and tells when the chosen programme/show starts and approximately when it ends. The learners then individually answer the questions related to the TV guide, which we check at the end. **Revision through movement:** timing different activities: runs (60 m, 300

	m, 600 m), training grounds (natural forms of movement, ball games, gymnastics), elementary games, etc. - timing, discussing the data - using simple stopwatches (on phones) for learners to time each other in pairs. ● Task 1: The learners work in pairs. They measure each other's times for the 60 m, 300 m and 600 m runs (the teacher chooses whether they will run 60 m and 300 m or 60 m and 600 m). They can use the stopwatches on their phones or regular stopwatches. The result (time) of the run in each event is recorded on the learner's personal sports chart (Appendix 5). ● Task 2: In case of bad weather, the activity can also be done in the gym. The teacher and the pupils will prepare a polygon/training ground? or training stations "Who is faster?" The exercises included in the stations can be: ○ Natural forms of movement and play - different forms of walking, running, elementary throws, jumping, climbing, crawling, rolling, etc.); ○ Ball games - keeping the ball in place with the hand, foot and stick, moving in a straight line and changing direction; hitting various stationary and moving targets while moving by rolling the ball, throwing (one and two-handed) and hitting the ball with the foot or stick, etc., ○ Athletics alphabet and ○ Gymnastic alphabet ... Instructions for the training ground: demonstrate the movement tasks on the training ground. Divide them into pairs. One partner performs the movement tasks on the training ground, while the other times him/her. The pupil who has the shortest time to complete all the movement tasks wins. The pupils can repeat the training ground several times and monitor whether they have improved their time. Instructions for the training stations: show the pupils the movement tasks at the stations. All the pupils need to perform the same number of repetitions (e.g., 15 or 20) at each station. Once the pupil has completed the required number of repetitions, he/she moves on to the next station. The first to complete or the one with the shortest time to complete all the required movement tasks at all stations wins. The time can be measured by either the teacher or the pupil himself/herself.
Extension activities	With the prepared material, we can carry out similar activities in higher classes by monitoring our progress in runs, creating various graphs, calculating progress in percentages.
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanič, Mojca Žefran, Tadeja Volmut (Department of Elementary school, University of Primorska)

Title	**Tessellation**
Key words	Tessellation, shapes, art
Short description	Tessellation is a pattern of repeating shapes without gaps or overlaps as they cover a surface or geometric plane. It's an important part of any 2D shape topic and is typically introduced to children from the age of 6 onwards. Tessellation develops students' spatial abilities
IT tools	
Fields (select)	A1: **Maths** A2: Natural sciences **A3: Art** A4: Music A5: PE A6: Mother tongue A7: Foreign language Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: tessellation, pattern
	A3: creativity
Expected prior knowledge	knowledge of geometric figures and the regularities of patterns
Expected learning outcomes	MATHEMATICS Learners: • learn about tessellation, • find tessellation in real word, • develop spatial thinking, • continue the pattern. ART Learners: • develop creativity through tessellation.
Expected duration	90 minutes
Preparation	The following should be prepared (see appendices): - samples of tessellations, - photographs of tessellations used in art and architecture, - scissors, models of patterns, - pencils, crayons, adhesive tape - paper.
Detailed description of activities	**1. WARM-UP** - Tell the students that today we are going to look at tessellation. Ask them if they know what the word tessellation/plaster means. - Discuss with the pupils where paving stones can be seen (e.g. in the courtyard, on walls, in the staircase, etc.) - show them the pictures in Appendix 1 - discuss the shapes of the paving stones used, e.g. square, large and small square, hexagon, square and triangle, etc. Students describe the pictures they see and tell if and where they have seen something similar. When describing the pictures, look for patterns, e.g. colour patterns that are characteristic of tessellation. We also emphasise that there is no empty space between the different shapes.

	2. MAIN PART  - Divide the students into groups. Give one group some square templates, one group some equilateral triangle templates and one group some equilateral hexagons (see examples in Appendix 2). - The students' task is to sketch a sign on an A4 sheet so that the sides of the signs meet and fill the whole area. The children then colour in the characters to create a pattern. - After the activity, the students present their products and we look together to see if they have met the criteria (no gaps between shapes, a completed sheet of paper and following the pattern). - Give each group of students coloured hexagons, squares and triangles to cut out. Make sure that the sides of each figure are the same length so that the students can trace the mosaic accurately. An example of a mosaic can be found in Appendix 3. Each group should be given at least 4 hexagons, 24 squares and 24 triangles - students can cut them out themselves using the model. Discuss with the students how they have filled the space and what are the characteristics of tessellation with these three shapes. - Can tessellation also be done with circles? And why not? **3. CONCLUSION:** **Making your own tessellation** Students cut out a 5x5 cm square. Instructions for students: Cut out part of the left side of the square and slide it directly onto the right side of the square. Cut out part of the bottom side of the square and slide it directly onto the top side of the square. For example: Once they have the tessellation shape ready, they wipe it on paper and then paint it to create the pattern. They then present their designs to their classmates. Examples of design: 
Extension activities	Example of tessellation on a computer: https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Tessellation-Creator/
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanić, Tadeja Volmut, Mojca Žefran (University of Primorska, Faculty of Education)

Title	**We learn to see**
Keywords	Geometric characters, traffic signs
Short description	As part of the activity, students will repeat already learned geometric figures, connect them with road signs and name them in English. Students should recognize the character's name and color and create a cardboard model of the character based on this. Through an interactive game and worksheets, students have to find given pairs of geometric figures and name them.
ICT tools included	
Areas (select)	**A1: Mathematics** **A2: Natural science** **A3: Art culture** A4: Musical culture A5: Physical culture A6: Mother tongue **A7: Foreign language** Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: Writing numbers and mathematical symbols; counting to 20
	A2: Traffic signs
	A3: drawing, cutting
	A7: naming characters and colors in English
Expected prior knowledge of students	Students know letters, numbers, and the basics of the English language
Expected outcomes	Recognition of geometric figures, recognition of traffic signs, safe movement on the road
Expected duration of the activity	45 minutes
Preparation of activities	1. Prepare thin cardboard and accessories for cutting and coloring 2. Prepare the game Memory - pairing 3. Prepare working materials for learning and recognizing traffic signs and their shapes.
Detailed description of all teaching activities	At the beginning of the activity, the teacher tells the students that today they will repeat the learned geometric figures and that they will try to recognize the shapes of certain road signs. In the first part, the teacher divides the students into groups, and each group takes a piece of paper from the box with the name and color of an individual geometric figure in English. Based on the recognition of the character and color, the character should be made of cardboard and painted. In the second part of the activity, with the interactive Memory game, students have to find given pairs of geometric figures. If the geometric figure of the overturned cards matches each other, the student must name the figure and determine its color in English. As long as the student succeeds in turning over a pair of cards that match each other, he has the right to turn the cards over. As soon as he turns two cards that do not form a pair, his right to turn cards is over. The student whose turn it is then tries to remember where that card is, so that he can turn over the

	corresponding pair. For the third part of the activity, prepare work material with traffic signs where students can see traffic signs and their names, and their task is to connect the corresponding traffic signs with the corresponding geometric figure. Discussion opens: *Which of these signs have you seen so far?* *Show me a sign that has the shape of a triangle!* *With which character could you associate the no-traffic sign on a bicycle that you see in the picture?* The final part includes homework instructions for students to find one thing in their home that reminds them of a character and describe it in a notebook (character, color, number of pages, size...)
Possibilities to expand activities	It is possible to prepare similar activities on the computer, through connection and recognition games as well as a crossword puzzle.
Additional notes	
Authors	Antea Čilić, Hrvoje Ljubić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Title Earth Day - 22.4./ Planet Earth, continents, countries**
Keywords	Planet Earth, continents, countries, maps
Short description	The aim of the activity is for the students to practice navigating the map through three activities, and to solve problems with four mathematical operations with numbers up to 1000
ICT tools included	Google Earth, globe, maps
Areas (select)	**A1: Mathematics** **A2: Natural science** A**3**: **Art culture** A4: Musical culture A5: Physical culture A6: Mother tongue A7: Foreign language Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: addition and subtraction of numbers up to 1000, multiplication, division, sets
	A2: Protection and preservation of the environment, finding your way on the map
	A3: coloring
Expected prior knowledge of students	Students know numbers, know basic mathematical operations with numbers up to 1000, they understand what is set, they are familiar with maps
Expected outcomes	Student: -multiply and divide within the multiplication table, -applies four arithmetic operations and relationships between numbers, - applies the rules in calculating numerical expressions with brackets, - navigates and interprets a geographical map, -adds and subtracts in the set of natural numbers up to 1000.
Expected duration of the activity	90 minutes
Preparation of activities	Prepare worksheets
Detailed description of all teaching activities	PART 1 (15 minutes) It is explained to the students that they live on planet Earth, how many continents it is divided into, which continent their country is on, and on a world map or globe (maybe Google Earth or something similar if the classroom is equipped) name the continents, and show the country where they live and name and show the neighboring countries. The students are instructed that Earth Day is celebrated on April 22, and that a short workshop on that topic will be held in class, in which the goal will be to repeat basic calculation operations and show how they navigate the map. -students are given worksheets with coloring pages - instruct them that the goal is to color the map of Europe correctly,

	whereby the number written in the figure representing a country determines what color that figure will be painted, and that through the first two activities, they will find out which number represents which color. Students are given worksheets for the first two activities and a sheet in which they will write the numbers they get in activity no. 2. PART 2 (10 minutes) Activity 1: -instruct the students that they will first need to color the flag of their country, and that they will find out what colors will be represented by the numbers written in the figures inside the flag in further activities. PART 3 (30 minutes) Activity 2: -instruct the students that they will first color the map of the world by continents, that they will find out which colors will be represented by the numbers written in the figures inside the flag in further activities. - with this map there is an index that connects the name of the continent and the color, while the figuresthat represent the continents are written with tasks that lead to the numbers that represent the colors for the next activity PART 4 (35 minutes) Activity 3. - students should color the map of Europe based on the previously obtained connection of colors and numbers - they need to recognize what color their country is painted -they need to conclude which countries are colored by the color represented by the number 45 (a set of EU member states) Repeat with the students in which place, which country, on which continent they live. How many countries does Europe have, and how many countries are adjacent to their country, does their country belong to a special group of countries (E.g. Eu)
Possibilities to expand activities	
Additional notes	
Authors	Antea Čilić, Mila Zovko (FPMOZ, University of Mostar)

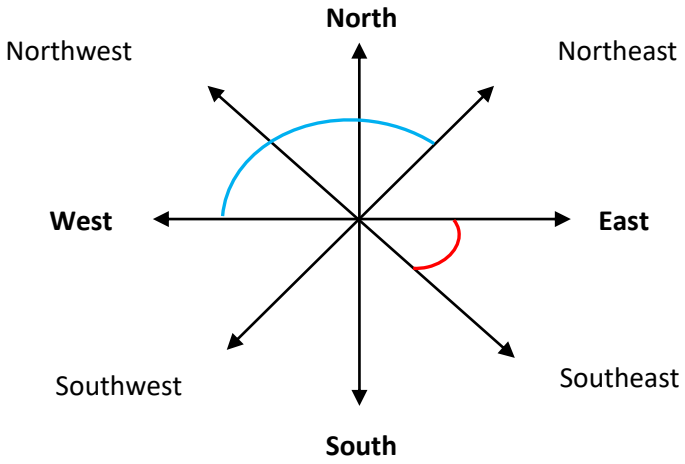
Title	**Earth Day - 22.4. Recycling**
Keywords	Recycling
Short description	The goal of this activity is that students repeat basic mathematical operations, sets, and to learn how recycle bio-waste by themselves through the preparation of fruit salad.
ICT tools included	Gallon, knives, plastic cups, cutting board, various fruits and a scale.
Areas (select)	**A1: Mathematics** **A2: Natural science** A3: Art culture A4: Musical culture A5: Physical culture A6: Mother tongue A7: Foreign language Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: units of measurement for mass (kg, gr..), addition, subtraction, division, sets, ratios (larger/smaller, faster/slower)
	A2: Protection and preservation of the environment, 3rd grade, Cleanliness of the environment, 2nd grade
Expected prior knowledge of students	Students know numbers, know basic mathematical operations with numbers up to 1000, know measurement units for mass, know the terms biowaste, recycling
Expected outcomes	Student: -applies four arithmetic operations and relationships between numbers, -expresses mass in different measurement units - knows how to dispose bio-waste and the importance of recycling
Expected duration of the activity	90 minutes
Preparation of activities	1. Prepare a larger gallon, for example for 5l water, plastic cups, knives and a cutting board, various fruits and a scale. 2. Students can bring a few pieces of fruit themselves or the teacher can prepare them in advance.
Detailed description of all teaching activities	PART 1 (15 min): Repeat the basics of recycling with a special emphasis on bio-waste and how students can use it themselves to produce compost to promote the growth of plants at home. PART 2 (60 min) With the guidance of the teacher, the students will count how many types of fruit they have that were brought to prepare the fruit salad, weigh how many kg/gr of each fruit they have, and based on this data, calculate how much fruit there is in total. Comment on which fruit they have the most, which one the least... Then they need to peel all the fruit, cut it into cubes in a larger container, weigh it again, comment on how heavy the leftovers from peeling the fruit are and how much fruit each of the students can have for a snack so that everyone had the same amount. This will repeat addition, subtraction and division.

	It is explained to the students how many kg/gr of fruit goes to peeling waste, and how we can use it for the garden. After the students have eaten the fruit salad, put all the leftovers from the fruit salad and fruit peeling into the gallon, explain why they should not put in, for example, bags of biscuits, cups of yogurt and the like, and refer to the fact that with almost every preparation part of the waste is bio-waste. After everything has been put into the gallon, the total amount of waste is weighed, the teacher takes a picture of what the waste looks like on the first day, and the goal will be for the students to monitor how long it takes for the waste to completely decompose, commenting every few days on how many changes have occurred on waste, when a fruit has decomposed, after how many days all the waste decomposes. PART 3 (15 minutes) At the end, when everything has decomposed, students can comment on which fruits decompose the fastest, which ones the slowest, etc. The resulting compost can be used to fertilize flowers in the school garden.
Possibilities to expand activities	1. It is possible to come up with a combination of this topic with the topic "Chemical kitchen" by colleague Marija Lesjak 2. In the "Little Gardeners" scenario, fertilize the garden with bio compost.
Additional notes	
Authors	Antea Čilić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Meet me at the corner**
Key words	settlement, streets, (non)parallel lines, angles
Short description	The scenario's central concept is based on utilizing the settlement plan's network of connected (non-)parallel lines and angles for movement through the area. In the countryside, or in their immediate surroundings, students can see how the layout of the streets makes certain angles, how they run parallel to one another, how they are normal to one another, and so on. Students are given instructions on how to get from their starting point to a predetermined destination with a classmate using mathematical vocabulary and the Google Earth/Maps/Streets applications (parallel, turn at right angles, etc.). It is discussed and concluded that the environment (including relief or large rivers, for example) influences the formation of a settlement's street network. This is a great way to integrate learning across the natural sciences and give students experience with spatial navigation at the same time.
ICT tools included	*Googel Earth/ Google Maps/Google Streets* Online free NSmaps platform for displaying the urban network of Novi Sad, with numerous layers of data that can be turned on and off as needed (http://www.mapanovisad.rs/mapserver2015/nsmaps/#) (https://novisad.com/mapa) *Programme Paint*
Fields/ subjects	A1: Mathematics A2: Nature and society *A8: Informatics (indirectly)*
Topics (for each selected subject)	A1: Parallel lines. Drawing parallel lines; Normal straight lines. Drawing normal lines; Angles. Types of angles. Drawing angles. A2: Orientation; Sides of the world; drawign scale; Settlement plan *A8*: *indirectly included - familiarization with the basic techniques of interactive online tools (search, zooming, information); drawing and coloring lines and polygons*
Expected prior knowledge of students	☞ *Elementary spatial navigation skills (sides of the world)* ☞ *Prior knowledge of lines and angles* ☞ *Elementary knowledge of colors, shades, drawing and coloring* ☞ *Basic computer skills*
Expected outcomes	☞ Improving the knowledge about lines and angles ☞ Recognition of parallel and normal lines, as well as angles in the local environment on the example of the street network ☞ Recognizing different types of street networks and connecting them with relief, waters and other features of the local environment ☞ Drawing paths (routes) using lines and certain angles in the settlement grid (on paper or using a computer)
Expected duration of the activity	2 school hours (90 minutes), with the possibility of extension to additional Art and i Informatics classes
Preparation of activities	Download a documents with a map of the settlement (if it is not possible to use a computer in the classroom) via link https://inamath.uniri.hr/meet-me-at-the-corner/
Detailed description of all teaching activities	**Activity 1: Testing and improving of knowledge of lines and angles Utvrđivanje linija i uglova – recognition and drawing exercise** The teacher goes over what parallel and normal lines are with the students, and they draw them on the board. Then they repeat what an angle is and what kinds of angles there are. They also draw angles on the board and in their notebooks.

Activity 2: Testingand improving the knowledge of orientation and sides of the world

The teacher draws or shows the sides of the world on the (smart) board, and then the students determine the relationship between the lines representing the sides of the world. After that, they recognize at what angle certain sides of the world cross each other. Students draw in their notebooks. The teacher first draws the primary sides of the world, then the students go to the board and draw the secondary sides of the world (the final scheme is shown below).



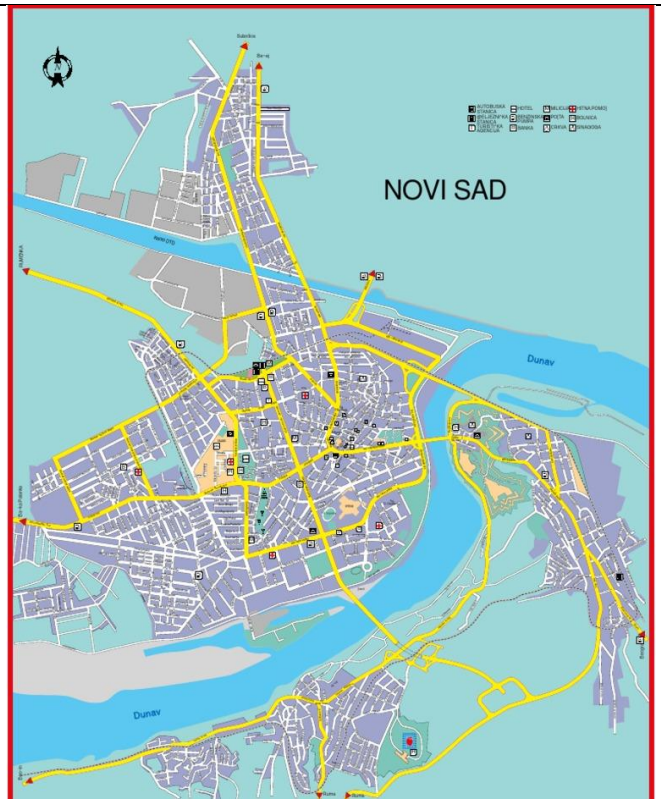
The lines, which show the sides of the world, are parallel to each other and cross each other at right angles. Draw lines to show the northeast, southeast, southwest, and northwest sides of the world.

Put red on the lines that point east and southeast. How do they fit together? The sketch only shows the corner, so students can color the whole line. Mark in blue the lines that show west and northeast? What angle do they form?

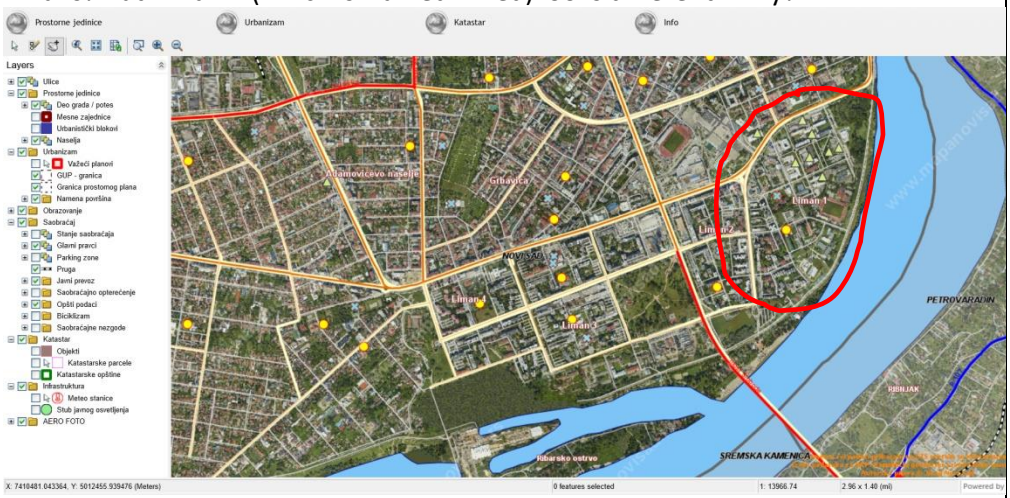
Activity 3: Connecting the lines,angles and sides of the world with street network and settlement map

The teacher goes over what the students have learned about settlements, how they look, and how the streets are laid out (plan and network of settlements). The teacher then shows examples of how the base of the settlement looks and talks with the students about it. Some towns were built along the river, either on one or both of its banks (Novi Sad, Sremska Mitrovica, and abac) (for example, Belgrade). Some towns were built in hilly places (on the mountains, numerous villages south of the Sava and Danube). In Vojvodina, a lot of settlements were built in the plain. The teacher shows the students on the smart board or prints out several neighborhood plans and gets the students to comment on the layout of the plans, using their knowledge of straight lines and angles.

Let's say, Novi Sad can be one example (the teacher can do any other example). What kind of angles do the streets in Novi Sad generally form? Are the streets parallel and normal to each other? (The image can be downloaded in a larger format, or enlarge it or choose another one).

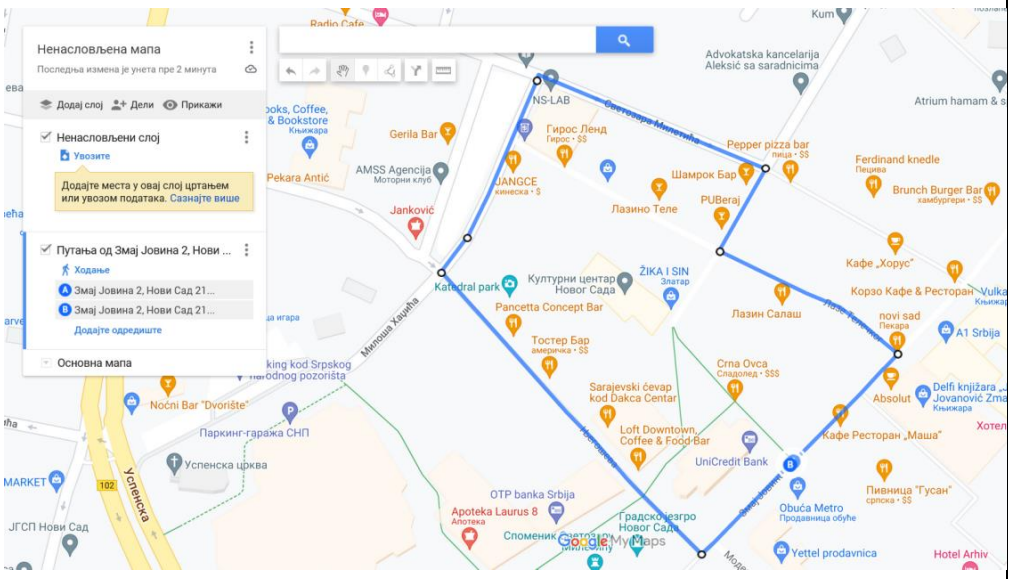


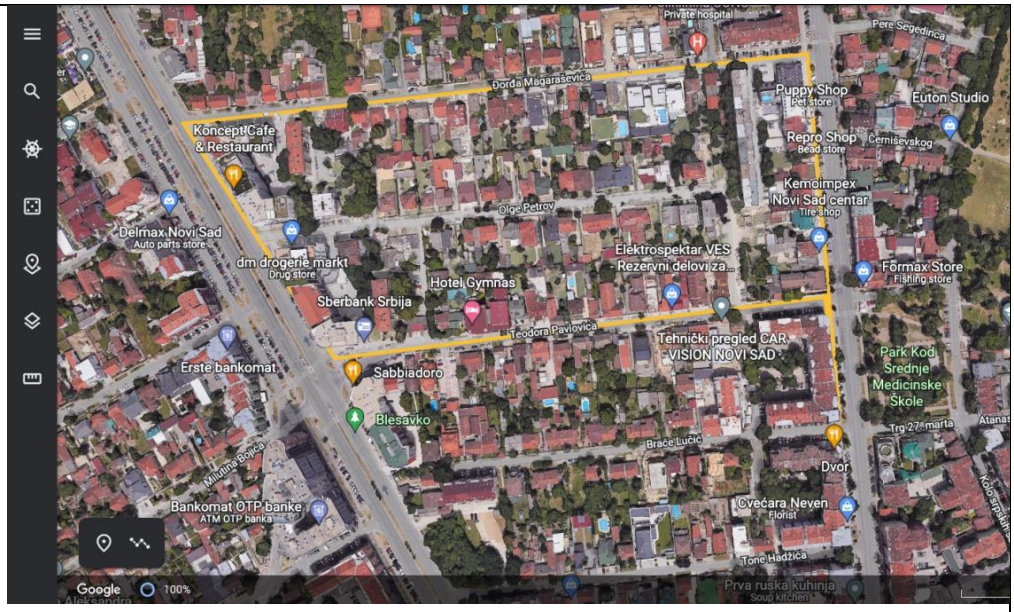
The teacher shows the students parts of Novi Sad on the NSmaps portal. They talk about how the streets look and the angles they make. You can turn on and off side layers, like traffic accidents. The teacher can turn on that layer and then zoom in on certain places where there are a lot of traffic accidents so that students can talk about why they are so dangerous. There are layers for parks, schools, and hospitals, so they can be seen separately based on the weather. Limans are one of the most well-known parts of Novi Sad. What are the street angles/corners like in Limans? But Liman 1 (which is framed in red) looks different. Why?



Activity 3: Drawing a path (a walking route for example) using knowledge of lines and angles and applying it to streets

The teacher shows the students how to use Google Maps (required to have a Google account). She/He types the name of a street into the search engine, and the students figure out the relations between straight lines and angles of all the streets with which it intersects. If they all have the same relations, then the streets

	are parallel. The teacher draws a walking path of their choice and the students comment on the relationship between the streets (lines).  The teacher prints out any part of the Novi Sad map, gives it to the students, and tells them what to do. Every student makes his or her own path and then talks about it. The same exercise can be done if the student has a computer with Google Earth or Google Maps in front of her/him and the teacher points on the smart board while the student draws the path. Students might be given a sheet with instructions and spaces to fill in. **Example:** *Jelena and Petar are located on the corner of Braće Lučić and Cara Dušana streets. They continue walking along Cara Dušana Street. They turn at _____angle into Teodora Pavlović Street. At the end of that street, at an _____angle, they turn northwest into Bulevar Evrope. Jelena and Petar continue their walk and at the _____ angle turn into Đorđe Magaraševića Street. At the end of that street, they decide to turn south and continue down Cara Dušana Street at the _____ angle to where they started.* *Draw of the path*



Activity 4: Connecting the gained knowledge with the local environment and natural conditions of settlement formation

The teacher shows the students sketches and plans of different towns on the smart board, and they talk about it. If the students have access to a computer, they can use Google Earth to find the neighborhood the teacher mentions and talk about it as a group. Why are the houses far apart, and why do some of the streets wind around? Why are most of the houses in Vojvodina close to each other, most of the streets cross at right angles, and the biggest boulevards are all connected? Does the relief change how the settlement looks? Does the river change how the settlement looks?
Some examples:

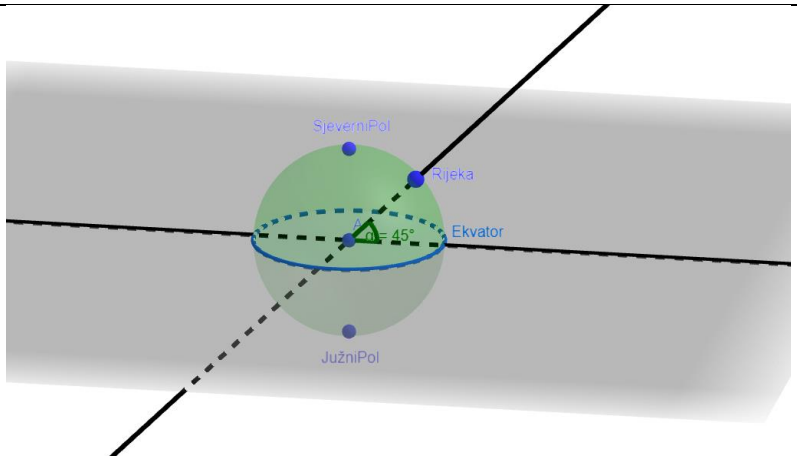
The village Marinovac, *southeastern Serbia* (Students can figure out on their own which side of the world it is, i.e. position in Serbia)

Nova Varoš, *southwestern Serbia* (Students can figure out on their own which side of the world it is, i.e. position in Serbia)

	 Belgrade and neighbouring settlements 
Possibilities for expansion of activities	**Possibility of connecting activities with Informatics** Basic computer skills and online platforms. Using the example of an online interactive map, students can practice zooming, panning the map, searching, drawing paths, and reading information about selected locations.
Additional notes	If a student attends classes according to the IOP1 or IOP2 program, it is necessary to include a personal companion/assistant in the preparation, and choose activities in which it is possible to include the student as much as possible.
Authors	**Milica Solarević, PhD, Associate Professor, UNS**

Title	**What's the point of this angle?**
Key words	Angle, protractor, measure of the angle, latitude, longitude, time, time measuring, sundial, Sun, Earth, seasons, gnomon
Short description	Within the activity of making a sundial, science and math content correlate. In constructing a sundial, students acquire geometry terms and concepts, such as angle, constructing, and measuring angles on paper. Students learn about the relation between seasons, times of day, and the angle at which sunrays fall on the earth's surface. In this topic they learn about the position of the Earth and the Sun, and about their position on Earth relative to the earth's axis and the Equator. Sundial is an excellent topic in which we can relate math with everyday life and demonstrate the importance of math, especially calculating the angle size in everyday life (in roof construction, installation of solar panels, planting vineyards on slopes, etc.)
IT tools	Geogebra
Fields (select)	**A1: Math** **A2: Science** A3: Art A4: Music A5: P.E. A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: angle, angle size, circle, time measuring
	A2: Sun, Earth, globe, north and south pole, latitude, maps, seasons, parts of day
Expected prior knowledge	Angle, right angle, obtuse angle, acute angle
Expected learning outcomes	• Determines and draws an angle. • The student recognizes the importance of organizing time and displays the timeline of events. • The student concludes about the organization of the local community, compares depictions of different spaces
Expected duration	45 + 90 minutes
Preparation of the activities	
Detailed description of activities	A pair of compasses, protractor, ruler Worksheet: How to measure an angle? Two balls (or a source of light and a globe) Cardboard or thick paper cut in the shape of a square, with sides lengths 12 cm and a stick or straw for each student (possibly of the same size as square sides) If necessary, teacher can read more on the topic of seasons (godišnja doba)

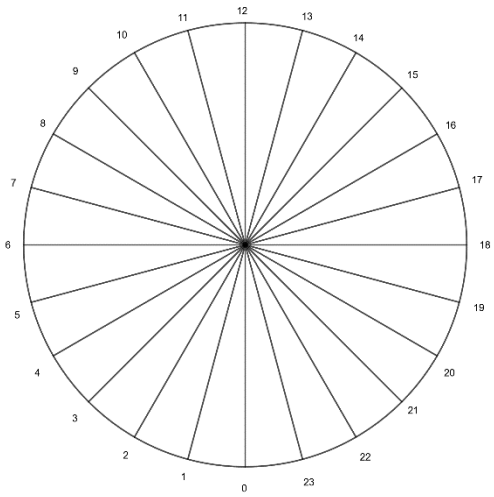
	1. How do we measure an angle? (45 minutes) Note: This activity can serve as a revision task, if students are already familiar with protractor and they already know how to measure the angle size. To carry out this activity students should be familiar with working with protractor. Students revise the definition of an angle and they list and describe various types of angles that they've learned so far (right angle, obtuse angle, acute angle, straight angle). Furthermore, students revise units of measurement they've learned so far and we introduce measurement units to measure angles. Teacher informs them that angles are measured in degrees and the tool to measure with is called a protractor. Students are introduced to a protractor. Teacher relates the angle size with some angles they've met so far. They are familiar with the right angle. Teacher informs them that right angle measures 90 degrees and they use a protractor to check. They conclude that the size of a straight angle is 180 degrees. To continue, they draw an angle that measures 270 degrees, and another one at 360 degrees. Students try to answer the following questions: What is the size of an obtuse angle? What is the size of an acute angle? Students do two more exercises, using a protractor. In one task, they draw arbitrary angles and measure their size. In another, they draw an angle that measures 60 degrees and another that measures 120 degrees. 2. How does Sun define our rhythm (15 minutes)? What are measurement units of time? Why does a year have 365 days and a day 24 hours? What happens in one day and what happens in one year? This activity describes the relation of Earth and Sun, and describes the position of the Earth in relation to the Sun in different parts of day and in different seasons. In each day, due to Earth rotation around its axis, the angle at which the Sun shines onto the Earth changes. That's why we distinguish day and night, before noon, noon and after noon. The teacher shows a demonstration. A student holds a laser and shines onto the Earth, while the teacher turns the globe around its axis. Similarly, teacher shows the movement of earth around the Sun in a year. One student is the Sun and shines onto the Earth while the Earth spins around the Sun. They observe the way the Sun shines on different parts of the world in different seasons. Teacher and students notice that different seasons are the result of the size of the angle at which the Sun (source of light) shines onto the earth (Europe, for example). Students observe what happens at the North pole. 3. Can the Sun be of help in defining the time? (60 minutes) It can! This is where we need one specific angle. Its name is latitude.



The design of Equatorial sundial

Step one: Students draw a circle, using a pair of compasses, 5 cm diameter (if the squares with 6cm sides are prepared), and mark one radius. Students already know that complete angle is 360 degrees. Furthermore, as they know that there are 24 hours in a day, they will divide 360 by 24 and get the solution that the angle between the longitudes that will define hours, is 15 degrees.

With the help of a protractor, they draw the following picture:



The picture represents a dial to measure time in summer. It is placed on a cardboard and we use it to tell time.

Note: To measure time in winter we need to read it on a dial on the bottom part of the square and the numbers are written counter clockwise.

Step two: Students make a hole in the middle of the square and in the middle of the circle cut out of paper and stuck onto the square, in such a way that 0,6,12,18, are at the midpoints of square sides.

Step three: In this step, students create a gnomon. A gnomon is a stick that casts a shadow. Each student gets a stick or a straw. They put the stick through holes, and it is positioned vertically to cardboard square. Gnomon must face the floor at the angle the size of which equals latitude of the current location.

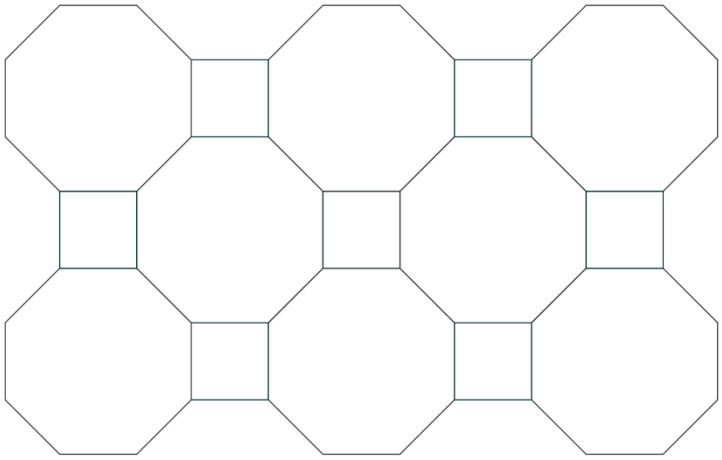
In our case, the angle is around 45 degrees and the gnomon length from the floor to the square equals half the square side. Considering that students lack the mathematical knowledge necessary to reach the conclusion, they can get to it through the experiment (legs of the right-angle triangle must be the same length), or the teacher can inform them that mathematicians can calculate the relation of the side lengths in a triangle, with angles of 45, 45 and 90 degrees.

	Therefore, the length of the part of the gnomon sticking towards the floor should be positioned carefully (if the square side length is 12 cm, then the part of the stick between dial and floor is 6 cm). 4. Find a place under the Sun (15 minutes) At this step we move outside the classroom. It is important to come out at the top of an hour (ideally at noon at daylight saving time). Students place their dial on the floor so the shadow of gnomon falls to the current time. We should take daylight saving time into consideration (Sun cannot tell winter or summer time, that's human agreement) and that in summer the Sun is at its highest around 1 pm. For the correct time, that would be in sync with our digital clocks, slight corrections should be made with regards to the latitude, but this will not be necessary in our case. We inform the students that there are longitudes at which shadows never will never fall (night time) and that at the times of equinox it is impossible to read the shadow on dial, because sun rays are parallel to the dial and on that day the shadow will cross from the top part to the bottom, or vice versa.
Extension activities	1. Having learned what a sundial is, how it works and how to make it, students can design a sundial in school yard or in school garden. Students can draw a horizontal sundial in chalk in the schoolyard. Students can use gardening constructions to draw a circle. This clock will show time even on the equinox, but it will not have a uniform division of angles, which makes this clock a bit more complicated to make. To calculate the size of angles to make a horizontal sundial, a free software can be used: https://www.shadowspro.com 2. Students will be given a story about time measurement as math reading, after the activity: https://www.skole.hr/mjerenje-vremena/ https://www.skole.hr/mjerenje-vremena-2/ 3. The story of angle can be related to solar panels that produce electrical energy. Angles at which panels are installed depend on the position of the Sun and the Earth. It would be ideal to change the angle of panels so that sun rays would fall vertically at any time. Similarly, the activity can be extended to the topic of roof building, vineyard planting on slopes, etc.
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

Title	**Math puzzles**
Key words	Polygons, tiling the plane, scientists and scientific research
Short description	This activity illustrates the process and follows the steps of scientific research. The research topic is tiling of the plane with regular polygons. Students aren't familiar with this term, but it is a notion simple enough for students to understand and use it in their own research. Except for the mathematical terms and concepts, this activity aims at helping students understand what a job of a scientist is, and to spark a positive attitude towards scientists, scientific facts and methods. It is possible to correlate this activity with civic education topics – especially ones that aim at creating a responsible, democratic society that makes decisions based on the available information. Tiling the plane is a great topic in which we can relate Math to everyday life and to Art. Students can do various tiling in Art classes, but also using computers in their IT classes.
IT tools	3d print
Fields (select)	**A1: Math** **A2: Science** **A3: Art** A4: Music A5: P.E. A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected)	A1: angle, triangle, quadrangle
	A2: significant scientists and scientific research
	A3: symmetry in artistic expressions
Expected prior knowledge	Angle, triangle, quadrangle, square, plane, flat surface, curved surface
Expected learning outcomes	• Describes and constructs a circle and its elements • He draws and constructs geometric figures. • It connects all known geometric shapes. • It compares the surfaces of the characters and measures them with unit squares. • Conducts simple research and analyzes the data obtained. • The student recognizes the importance of organizing time and displays the timeline of events. • The student displays the timeline of events and assesses their importance • The student interprets various contents with artistic and visual expression. • In his or her own work, the student uses the technical and expressive capabilities of new media technologies
Expected duration	90 + 45 minutes
Preparation of activities	1. If necessary, teacher can study the topic of tiling the plane in more detail: (http://e.math.hr/old/poplocavanja/index-print.html) 2. Prepare various regular and irregular polygons. It is important that every

	regular polygon has a side of the same length. Regular polygons can be printed on a regular printer, cut out, or printed in 3D printer. These polygons are necessary for one group of students: • at least 3 dodecagons (white) • at least 9 octagons (green) • at least 6 hexagons (yellow) • at least 20 quadrangles (red) • at least 50 triangles (blue) • 3 regular pentagons (black) • 3 regular heptagons (black) • 3 different irregular polygons (black) We attach .ggb files of Archimedean tiling. The attached files can serve as a preparation for coloring pages, or we can cut out regular polygons out of them. Polygon sides are all 4 cm longs (while printing the files choose scale 1 unit = 1 cm, if you wish to print polygons with sides 4 cm long): https://www.geogebra.org/m/kxwmn7jb https://www.geogebra.org/m/regamqzf https://www.geogebra.org/m/b49pttwq https://www.geogebra.org/m/ukreh4qw https://www.geogebra.org/m/fyjj9v4v https://www.geogebra.org/m/z7u6qfwy https://www.geogebra.org/m/nrjzf4gg https://www.geogebra.org/m/k8unubcx If you wish to use ready-made GeoGebra files to print tiles, we advise you to color the congruent polygons in the same color, because this way Archimedean tiling will look much nicer. If possible, you can print the polygons on thicker paper. Also, model (.stl or .obj) is available on Tinkercad. https://www.tinkercad.com/codeblocks/ebRpodNa11r https://www.tinkercad.com/codeblocks/66zm6bbCBXx https://www.tinkercad.com/codeblocks/ketohdkuW5o https://www.tinkercad.com/codeblocks/gDiHe4ZMFqW https://www.tinkercad.com/codeblocks/jlyx7552ew0 https://www.tinkercad.com/codeblocks/6PpnKPk2L0e https://www.tinkercad.com/codeblocks/gBvcia73PK7 https://www.tinkercad.com/codeblocks/cnOZtQXTyE3 https://www.tinkercad.com/codeblocks/4a3BNrVN1Gc If a magnet board is available, the activity of sticking magnetic strips on the board could present tiling. In this case this could be carried out with one set of polygons (i.e. students don't have to work in groups) 3. Teacher researches and finds one mathematician (preferably, a geometrician) from the students' homeland. 4. Worksheet (https://inamath.uniri.hr/wp-content/uploads/2022/11/Matematicke-puzzle-radni-listic.ej.docx)
Detailed description activities	1. Motivation (10 minutes) Teacher connects Science topics to the stories about significant scientists from their country. The teacher chooses one scientist, a mathematician preferably, and tells a story about his / her life. In our case, we will tell a story about Marin Getaldić (http://e.math.hr/math_e_article/br15/bilic_vlajsovic/hrvatskimatematicari). Marin Getaldić can be the main character of today's lesson, a scientist who conducts today's research. What do scientists do? Do scientists do an important job? It is a scientist's job to ask questions, to explore, to find the solutions to everyday life problems, to teach... Each scientist is a tiny gear wheel in a big system. Each wheel

	moves when influenced by other wheels, and thus they each move other wheels too... Science and scientific results have an immeasurable influence on our lives, on the quality of our lives and on our future. This influence is visible in all areas of our lives. What does a scientist's job look like? Do you know a scientist? Would you like to be a scientist? What is scientific research? Would you conduct scientific research? After this activity, students will be familiar with a scientist's job and with characteristics of scientific research. The theme of today's scientific research is Math, more precise: tiling the plane with regular polygons. 2. Scientists learn all the time. (1st step in scientific research) (15 minutes) (15 minutes) Each scientific research starts with learning and so will ours. The theme of our research is tiling the plane with regular polygons. We ask students questions and try to motivate them to think about the correct definitions: What do you think tiling the plane is? What does this term mean? What is a plane? What does tiling the plane mean? What is a polygon? Let's start from the beginning. What is a plane? A plane is a flat surface. How do we recognize a plane, what are its characteristics? A plane is not a point, it's not a line, and it's not space. But it is made up of points and lines and it extends in space. And how can we check if a plane is a flat surface? If we choose two points in the plane, a line defined by these two points must be a part of the plane. This can be illustrated with an example of a sphere or some other surface that is not a plane. Tiling a plane means finding a set of geometric shapes which have no common interior points, but the unity of which is the whole plane itself. The teacher asks whether students have ever seen the tiling of a part of a plane anywhere. He shows pictures showing the tiling of floors, walls, etc. And points out that people have always tiled walls and floors to decorate their living space. In our scientific research, we focus on special type of tiles, Math tiles, i.e., polygons, especially regular ones. A regular polygon is a new term for students and the teacher asks questions to elicit the definition of the term polygon. What is an angle? What is a triangle? What is a quadrangle? What is a polygon? The teacher shows pictures or models of various polygons and counts their sides and angles. Teacher leads the students towards the term a regular polygon. He/she shows pictures or models of various polygons and asks which of them would be called regular polygons. They reach the conclusion together – regular polygons are polygons in which the sides are all the same length and angles are congruent. Activity: teacher hands a bag with polygons to each group of students. The first task is to find and separate polygons that are not regular. If this is the first time that a student uses these props, they should be allowed to play with them and examine them, for at least 5 to 10 minutes. The teacher completes the activity and revises once more what the tiling of the plane means and what regular polygons are. At this stage students can draw several regular polygons and several irregular polygons, and at least one tiling of a part of the plane (of paper or a part of a paper). 3. Curious scientists ask a lot of questions (15 minutes) (2nd step in scientific research) The following step in scientific research is the quest for problems that need to be solved, asking questions that nobody has ever answered, etc. Scientists are curious and not only that their curiosity often makes the world go round but it also brings unbelievable discoveries. The teacher asks questions regarding tiling the plane with regular polygons. We've often seen floors tiled with square tiles, squares with sides 33 cm long, to be

	precise. Isn't that a bit boring? Can we use tiles in the shape of any other regular polygon, but keeping in mind that two tiles either should not touch, that they have a common vertex or that they have a common side. These tilings are called regular tilings. Let's think about how a mathematician would formulate the question. 1) Which regular polygons can be used (except squares) to tile the plane so that every two polygons have sides of the same length and so that two polygons don't touch, have a common vertex, or have one common side? If our tiler is very handy and we can describe precisely what we want, our floor might resemble the planes a famous Greek mathematician who liked to tile. In his honor, these tilings are called Archimedean tilings. We still must keep an eye on the same rules, but now our tiles can be different regular polygons (with all the sides of the same length). However, the tiling cannot be arbitrary. We must make sure that the situation around each vertex is the same. What does that mean? If we draw all the polygons clockwise around one vertex, then this sequence should not change (if we start in the same way). For ex. we start with a polygon with the smallest number of vertex. Teacher uses an example to explain.  For each vertex in the picture, we can draw the following: a quadrangle, octagon, octagon or just (4,8,8). That's why the picture shows one part of one Archimedean tiling. Advice: it can be demanding for students to acquire the way of writing the sequence made in Archimedean tiling, so in this part of the activity a presentation or animation can be made in which polygons would be colored while making a sequence around each vertex, as is shown in the picture. Students notice that tilings described earlier (with regular polygons with equal number of sides) are also Archimedean tilings. After students are introduced to Archimedean tiling, we can ask another question. What is the shape of the tiles we should buy if we wish for the tiler to lay them in Archimedean tiling? How many ways are there for the tiler to lay the tiles on our floor? Here we notice that all polygons have sides of the same length. A mathematician would formulate the question in the following way 2) How many versions of Archimedean tiling are there? 4. Scientists love to play. (3rd step: experiment in scientific research) (20 minutes) Experimenting is a very important step in scientific research because it is through experimenting that we sense certain behaviors, we notice regularities, relations, etc. Scientist form hypothesis based on experiments and they guess what the answer to the question from 2nd step might be.

	Three experiments are carried out: 1) Students hold “tiles” and try to guess how many versions of Archimedean tiling are there, in which all the polygons are congruent. Each student will easily arrange 3 tilings (with equilateral triangles, squares, and regular hexagons). 2) Taking the “tiles” (polygons) in their hands, students will try to guess how many versions of Archimedean tiling are there, in which all the polygons aren’t congruent. This task is more demanding for students. The teacher monitors the situation, and leads, and helps students with arranging all 8 tilings. The student who discovers Archimedean tiling writes the sequence on the board. Students count the discovered versions of tiling, and they sense those are all the tilings there are. 3) Teacher circles Archimedean tilings and asks students about the tilings that are not circled and why they aren’t circled. Students try to reproduce the results (false ones and true ones) that other groups of students got. They must arrange Archimedean tiling according to the marks on the board. This way important characteristics of a scientific experiment are pointed out to the students. Each result of scientific research must have the ability to be replicated. 5. Light at the end of the tunnel (4^{th} step: a confirmation of the hypothesis) (10 minutes) The most important step in scientific research is the confirmation of the hypothesis, i.e. the confirmation of the proof that the answer we sensed to be correct is affirmed. This is the key step in scientific research because without it, it would be impossible to implement the results and solve everyday problems. There are many rules that scientists must abide by in this step for scientific research to be valid, acknowledged, and applicable. This step is also the step that distinguishes scientific facts from non-scientific claims, the truth from something that cannot be proven. Mathematicians have proved that our premonitions are true and that there are exactly eleven Archimedean tilings, three of which are tilings with polygons with sides of the length. To conclude, the teacher returns to the tiler’s job and answers the questions. At this point, the teacher hands out a worksheet, students revise the terms they’ve learned, and they write their answers to the questions they posed in the scientific research. 6. Escher tiling (20 minutes) So far, we’ve discussed tilings with regular polygons. However, more interesting tilings are the ones with “unusual” tiles, the ones that cannot be bought in stores. However, with little mathematical knowledge they can be easily made. A famous Dutch artist Escher used Math for his “tiles”. The teacher shows some of the Escher pictures found on the internet. The teacher introduces the procedure to students to make their Escher tiling. 1) Take a square shaped piece of paper. 2) Draw a curved line, with the same starting points as one side of a square 3) Cut out the part of the paper bounded by a curved line and square side, with the same starting points as the curved line. 4) Stick the paper that you’ve cut along the opposite side of the one you drew. 5) If you wish, repeat steps 2,3 and 4 along one of the remaining sides of the square.

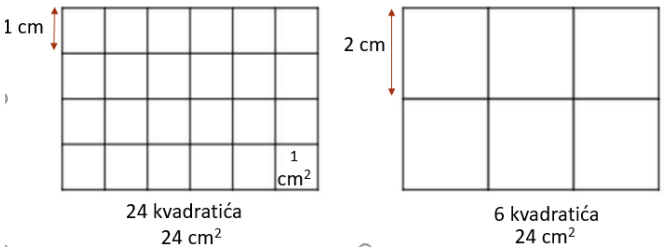
	It takes 20 minutes for each student to make his/her tile which is necessary to "tile"an A4 sheet of paper. Students can do it for homework or in the following Art class. If the students are decorating "tiles" after they've completed the paper, it would be a good idea to mention that they should decorate each tile in the same way. In this way they will paint Escher tiling. You can look forward to pieces of art made by your students!
Extension activities	1. Students can be involved in the preparation of the activity in several ways: 1) Research (by searching the Internet or by visiting the school library) on famous mathematicians (and scientists) from their country. 2) Drawing a polygon in GeoGebra in IT class. They will use this polygon in the activities in class. 3) Making of 3d model of polygon in Tinkercad and 3D printing in IT class. 2. After the activity in class, students can (within IT class) draw or arrange Archimedean tiling in: program GeoGebra, program Tinkercad, program Logo 3. If the teacher estimates that he / she will lack the time to complete the activity in class, it can be carried through distant teaching, as a virtual workshop. Virtual workshop is available in Croatian and in English on the following link: https://mod.srce.hr/course/view.php?id=349 4. Students can design interesting tilings in free programs: Mornaments (https://www.imaginary.org/program/morenaments) EcherSketch (https://eschersket.ch/) 5. Teacher can motivate students for drawing (by hand or using the computer) and organize a math exhibition at school. More details for the Math exhibition can be found in the article *How to organize a Math exhibition* – exhibition Taxi geometry (http://mis.element.hr/list/30/broj/113/clanak/1547/kako-organizirati-matematicku-izlozbu-izlozba-taxi-geometrija) 6. This topic is suitable for young learners as well, even for pre schoolers. Except for doing the puzzles with certain rules (Archimedean tiling), we can prepare A4 coloring pages with Archimedean tiling (printed black and white Archimedean tiling). They color and follow the rule that all congruent polygons are colored in the same way. 7. When teaching calculating of square area, teacher can assign the task to use Archimedean tiling to cover a shape and to use it to calculate the surface area. Similarly, students can calculate square area of a tile they need to buy if they want to tile the floor in Archimedean tiling. 8. In extra math classes, students can prove that number of regular tiling equals 3 (proof is simple and described in http://e.math.hr/old/poplocavanja/index-print.html).
Additional notes	
Authors	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

Title	**Scratch the surface**
Key words	Area of a rectangle, handball court, area measurement units, estimation of an area
Short description	In this activity, we introduce the following terms: surface of a geometrical shape, formula to calculate the area of a rectangle and of a square, and area measurement units. Fractals are introduced, and in this way, not only do students practice the calculations of area, but also algorithmic thinking and focus on following the steps of the process, by following precise instructions. By connecting P.E. and Math and by using the introduced terms and concepts, the surface of a part of the playground is estimated and the implementation of Math in everyday life is presented. This step aims at responsible management of the school property. This activity can be continued in IT class.
IT tools	GeoGebra
Fields (select)	**A1: Math** A2: Science A3: Art A4: Music **A5: P.E.** A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: area, area measurement unit, formula for calculation of rectangle and square area
	A5: handball and basketball court
Expected prior knowledge	Measurement units for line segment length, multiplication of multi-digit numbers
Expected learning outcomes	• Draws and constructs geometric figures. • Connects all known geometric figures. • Compares the surfaces of the figures and measures them with unit squares. • Conducts simple research and analyses the obtained data.
Expected duration	90 + 90 minutes
Preparation	Worksheet: Scratch the surface (https://inamath.uniri.hr/wp-content/uploads/2022/11/Zagrebi-po-povrsini_radni-listicv2.docx) Worksheet: Basketball A, B, C (https://inamath.uniri.hr/wp-content/uploads/2022/11/RukometnoZagrebi-po-povrsini_radni-listicAB-1.docx, https://inamath.uniri.hr/wp-content/uploads/2022/11/RukometnoZagrebi-po-povrsini_radni-listicCdio-1.docx , https://inamath.uniri.hr/wp-content/uploads/2022/11/RukometnoZagrebi-po-povrsini_radni-listicDE-1.docx) Worksheet: Handball A, B, C (https://inamath.uniri.hr/wp-content/uploads/2022/11/Zagrebi-po-povrsini_radni-listicA-kosarka-1.docx, https://inamath.uniri.hr/wp-content/uploads/2022/11/Zagrebi-po-povrsini_radni-listicB-kosarka1-1.docx , https://inamath.uniri.hr/wp-content/uploads/2022/11/Zagrebi-po-povrsini_radni-listicC-kosarka-2.docx) Worksheet: Scratch the surface end (https://inamath.uniri.hr/wp-content/uploads/2022/11/zagrebi-po-povrsini_radni-listic_kraj.docx)
Detailed description of all activities	1st part: Introduction of the term area (90 minutes) 1. INTRODUCTION (10 minutes)

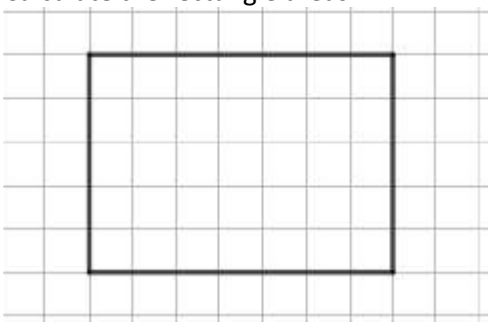
In introductory part we revise what area is and we point out the importance of calculation and measurement of area. We also note the situations in which these calculations are necessary (ex. Tiling, painting walls, buying table cloths).
Area is a number which we use to describe the size of the space taken by a plane figure (on paper, for example)

2. HOW DO WE MEASURE AREA OF A RECTANGLE? (15 minutes)

This part of the activity is done with worksheet Scratch the surface.
We show the following picture to the students and ask whether squares have the same area.



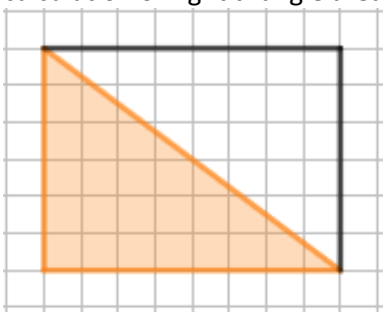
Students conclude that rectangles have the same area and that we can measure their area in square centimeters.
Through analyses of the picture, we get the formula for rectangle area ($P=a\cdot b$, where a and b are rectangle side lengths) and formula for square area ($P=a\cdot a$, where a is square side length). Square being a special type of rectangle.
Furthermore, students are informed that the choice of the units to express area depends on side lengths. This is presented in the following task.
Calculate the rectangle areas:



Side length of the unit square is half a centimeter. Here students conclude that side lengths will be measured in millimeters and area will be measured in square millimeters.

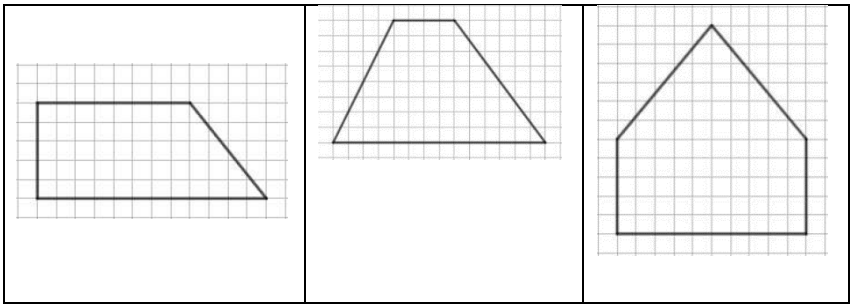
3. CALCULATING THE AREA (20 minutes)

At this point, students are handed pictures of plane shapes, the areas of which can be calculated by using the formula for rectangle area. In the first example calculation of right triangle area is calculated:



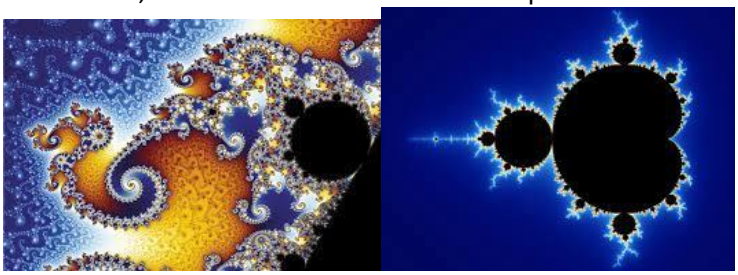
Students conclude that area is equal to a half of the complemented rectangle.
Furthermore, students are given pictures of the following plane shapes, with the instruction to divide the shapes to smaller shapes, the areas of which they can calculate (shapes are drawn in the grid in which a unit square has side length of 5

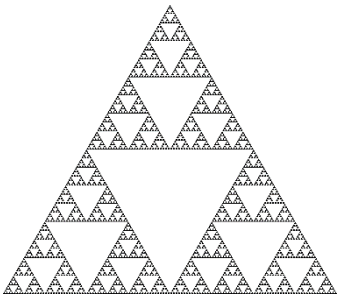
mm.)

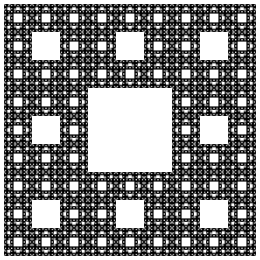


4. FRACTALS AND AREA (45 minutes)

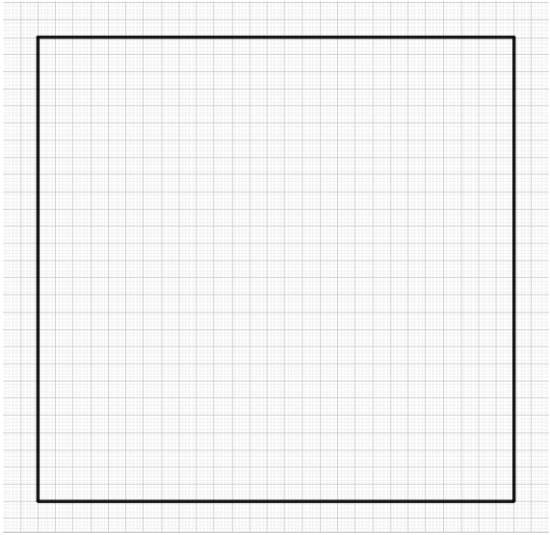
In the second part, we introduce the term fractal as a mathematical structure which is self-similar, i.e. the structure in which each part resembles the whole structure.



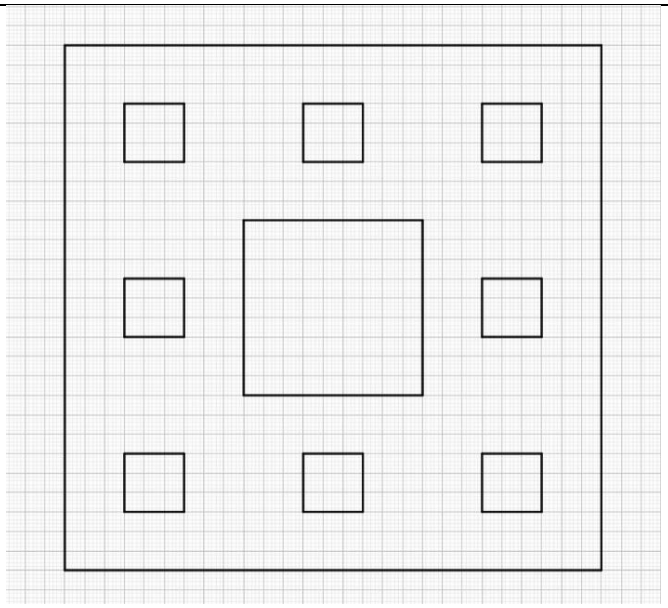




In the following part, students draw a fractal in fourth picture (Sierpiński carpet) on the paper with a square which is a union of 27 times 27 squares with side length 5 mm.



Students draw two iterations, with the help from the teacher.

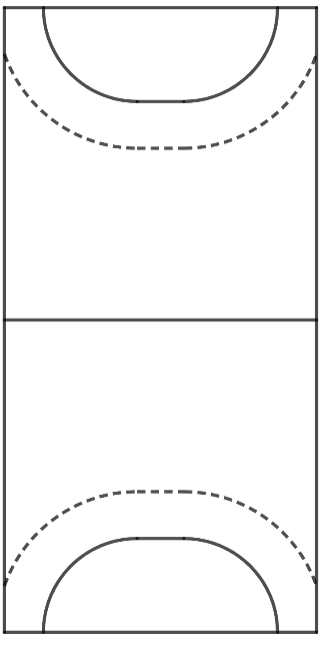


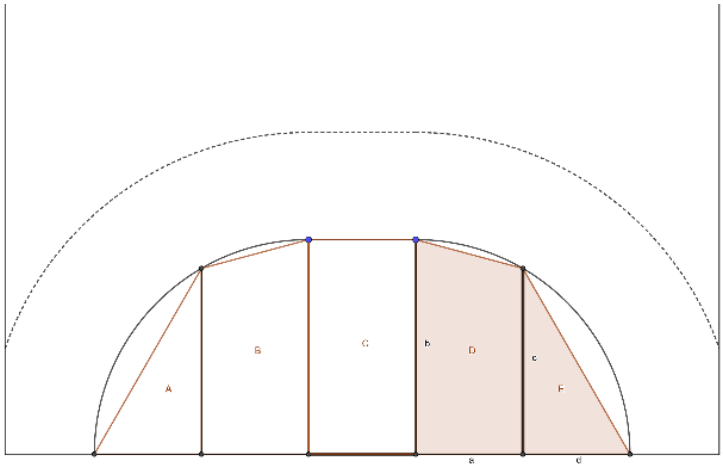
Students who are more skillful in drawing can draw the third iteration.
In the end of the activity, students calculate the area of the shape that was created after „excluding" the squares from the first two iterations. This is done by calculating the area of the starting square and subtracting the areas of the „cut" squares. If necessary, students can cut out squares from paper to understand the process of square creation more clearly.

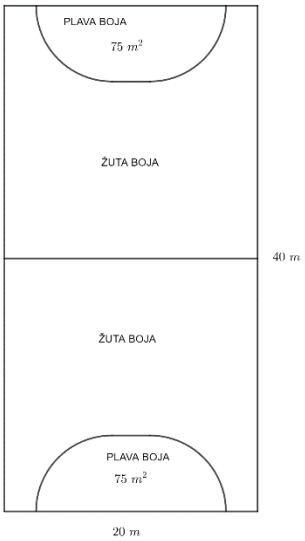
2nd PART: Calculation of area of curvilinear figure (90 minutes)

1. IN THE PLAYGROUND (45 minutes)

In the introductory part, we describe a sport (handball or basketball) to our students. We give information on the number of players, basic rules, and court. Furthermore, we motivate students with everyday problems, for example, the forbidden zone in the handball court is marked in different colors, to make it easier for the players to avoid that area. In this situation, it is important to be able to estimate the area of the part of the court to plan our expenses.
Students work in groups, and each group must calculate the area of a part of handball or basketball court. Within this scenario we suggest calculation of the area within 6 meters of the handball court and within 3-pointer in basketball court.



	In our scenario students work in groups of six. Each group has their task, for ex. the part of the court that must be measured is divided into three parts and one worksheet is allocated to a pair of students. Students are given the instructions to measure with their feet (always the same student) and write down the values  Advice: If the teacher finds it doable, children can distribute the assignments among themselves and split the court into parts they will measure. This is an option for 5th – 8th-grade students. 2. RETURN TO THE CLASSROOM AND CALCULATE WITH VALUES OBTAINED (30 minutes) In this part, students return to their classrooms and to their starting groups. They define the approximate value of the part of the court that they measured. First, students measure the feet length (in cm) of the student used as measuring tool. Then, they use that information to define side length in cm and calculate the area in square centimeters. They come to conclusion: 1 m2= 10 000 cm2 so they express the calculated value in square meters. With the help of GeoGebra files (https://inamath.uniri.hr/wp-content/uploads/2022/11/tocna-povrsina-rukometno-igraliste.ggb and https://inamath.uniri.hr/wp-content/uploads/2022/11/tocna-povrsina-kosarkasko-igraliste.ggb) teacher shows the real area, as well as the deviation. 3. CALCULATING THE STARTING TASK (10 minutes) One tin of paint covers an area of approximately 10 m2. How many tins of paint do we have to buy if we want to paint the area of a handball court as is shown in the picture? (worksheet)

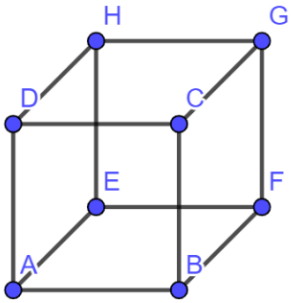
	
Extension activities:	1. In IT class students draw fractals using the computer, for example in GeoGebra, Tinkercad, Logo. 2. Project task: Approximate calculation of the area of a garden, a lawn, parking lot, yard, etc. Based on what they learned, students pick and draw shapes the areas of which they know how to calculate. To help them in this task, they will calculate the area of curvilinear shapes.
Additional notes:	
Authors:	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

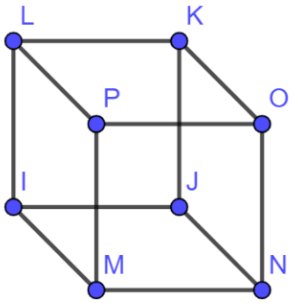
Title	**Who will produce less waste?**
Key words	Collecting and presenting the data, measurement units for mass, calculating to million, trash and waste, recycling
Short description	This activity connects Math topics with Science and mother tongue, but it also includes project research tasks that students do on their own. Except for acquiring Math terms and concepts, this activity aims at bringing the importance of recycling to awareness. To do this the activity uses specific data and calculations of the amounts of recyclable waste. This activity can be related to civic education curriculum which aims at encouraging responsible behavior of all individuals but also the need for responsible community managements. The activity can be extended to IT and foreign language classes.
IT tools	
Fields (select)	**A1: Math** **A2: Science** A3: Art A4: Music A5: P.E. **A6: Mother tongue** A7: Foreign language Other: A8: IT **A9: Civic education**
Themes (for each chosen field)	A1: calculating to 1000000, collecting and analysis of data, measurement units for mass, number of days in a year
	A2: waste, recycling
	A6: making posters
Expected prior knowledge	Calculating with big numbers, units to measure mass, trash and waste, recycling
Expected outcomes	• It uses natural numbers up to a million. • Adds and subtracts in a set of natural numbers up to a million • Multiplies and divides by two-digit numbers in a set of natural numbers up to a million • Applies four computational operations and relationships between numbers in problem situations • Conducts simple research and analyses the obtained data • The student concludes about the organization of the human body and life communities. • The student values the importance of a responsible attitude towards himself, others and nature. • The student analyzes and connects living conditions and diversity of living beings in different habitats and describes cycles in nature. • The student explains the results of his or her own research into nature, natural and/or social phenomena and/or different sources of information. • The student reads the text and retells the contents of the text

	using notes. • The student extracts important data using different age-appropriate sources.
Expected duration	90 minutes
Preparation of the activities	Worksheet (https://inamath.uniri.hr/wp-content/uploads/2022/11/Tko-ce-napraviti-manje-smeca-radni-listic-v2.ej.docx) Diary (https://inamath.uniri.hr/wp-content/uploads/2022/11/dnevnik-otpada-v2.ej..docx) Find and prepare informations about waste and trash for your local area.
Detailed description of all the activities	1. INTRODUCTION (20 minutes): revision of terms the students learned, related to waste and trash. Explain the data from table and discuss other types of waste which are not presented in the table, as well as the options for disposing of such waste. Video about three bottles. https://www.youtube.com/watch?v=_6xlNyWPpB8 2. DATA ANALYSES (45 minutes) In this part, students do the worksheet *Who will produce less waste?* 3. CONCLUSION (10 minutes): In the final part, we analyze the numbers we got and the importance of our influence on the reduction of the amount of waste, and how much we contribute to a better environment. Students make a poster showing the research data and point out the significance of recycling. This activity enabled the students to get insight into how much waste can be recycled in a month. At this point, they are assigned a research task in which they compare the amounts of recycled waste from their household to the possible amounts of recycled waste. 4. INSTRUCTIONS FOR RESEARCH PAPER (10 minutes): Students are handed instructions, a diary of waste and calculating tasks about their household.
	1. In Math class, students make a poster in a digital tool for making posters. 2. Students prepare a short text for the school paper, in which they present the numbers and explain the importance of recycling. 3. This topic can be extended to the foreign language classes where they would learn the terms that have to do with big numbers and types of waste.
Dodatne napomene	
Autori	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

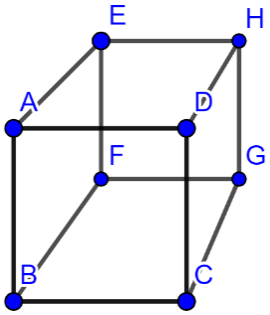
Title	**Drawing with Math**
Key words	Cube, cuboid, edge, intersection, parallel lines, drawing, rectangle construction
Short description	Within this activity, in drawing three-dimensional objects on two-dimensional paper Art and Math topics are related. By presenting a cube on a piece of paper, and using the rules of perspective drawing, students acquire a very simple painting technique and revise geometry content, such as intersection, edge, parallel lines, and rectangle construction with ruler and a compass. Students draw math objects and identify characteristics of each object. Students use perspective view of a cube to get outlines of houses in display of a street. Each student decides on their own about creative decorations of the houses. This activity can be continued in IT class.
IT tools	GeoGebra
Fields (select)	**A1: Math** A2: Science **A3: Art** A4: Music A5: P.E. A6: Mother tongue A7: Foreign language Other: A8: IT A9: Civic education
Themes (for each selected field)	A1: lines, intersecting lines, parallel lines, plane shapes, rectangle construction
	A3: display of objects in space
Expected duration	45+90 minutes
Expected prior knowledge	Point, length, line, half line, rectangle, parallel lines, intersection, cuboid, rectangle construction with geometry set
Expected learning outcomes	• Describes and draws a point, line segment, ray and line and their relations • Connects all known geometric shapes. • Compares the surfaces of the figures and measures them with unit squares • The student interprets various contents with artistic and visual expression.
Extension activities:	1. If necessary, teacher study the topic of perspective drawing in more depth: (https://inamath.uniri.hr/wp-content/uploads/2022/02/Perspektivno_crtanje.pdf) 2. Teacher prepares and provides photos of streets, buildings, etc. To be analyzed in class with students to introduce the rules of drawing three-dimensional objects on two-dimensional paper. 3. Before the activity students are assigned a homework task – try to draw a street as seen from its the top end (for. ex. The street where their school is, main street in their town, etc.) and try to find a photo of a street.

Additional notes:	1. Motivation (5 minutes) Analyses of the photos the teacher and students prepared. Photos show plane shapes, rooms, and streets. Students and teacher detect the way to make the impression of „depth", how houses and the street look, and how near houses seem, as opposed to those that are far. We notice that the objects that are near are bigger and those that are far are smaller. Even in reality the street is the same width in all parts, in the photo we get the impression it narrows.  Stradun (Dubrovnik)  Pelješac bridge To motivate students, pictures of solids drawn by Leonardo da Vinci are shown. (In these pictures it is clear that the „front" sides appear bigger than the back sides). It can be remarked that the rules of drawing to be shown today, are the rules that were used by Leonardo da Vince himself. 2. Presentation of a cube, on paper (5 minutes) Teacher shows a cube and pictures of cubes and asks which side of cube appears closer.





1st question: Which side of the left cube is closer: ABCD or EFGH?
2nd question: Which side of the right cube is closer: MNOP or IJKL?
We notice that from presentation we cannot conclude which side is closer.
Teacher shows the picture of cube drawn according to the rules of perspective drawing and asks the question:
3rd question: Which side of the cube is closer: ABCD or EFGH?



We expect all students to give the same answer.

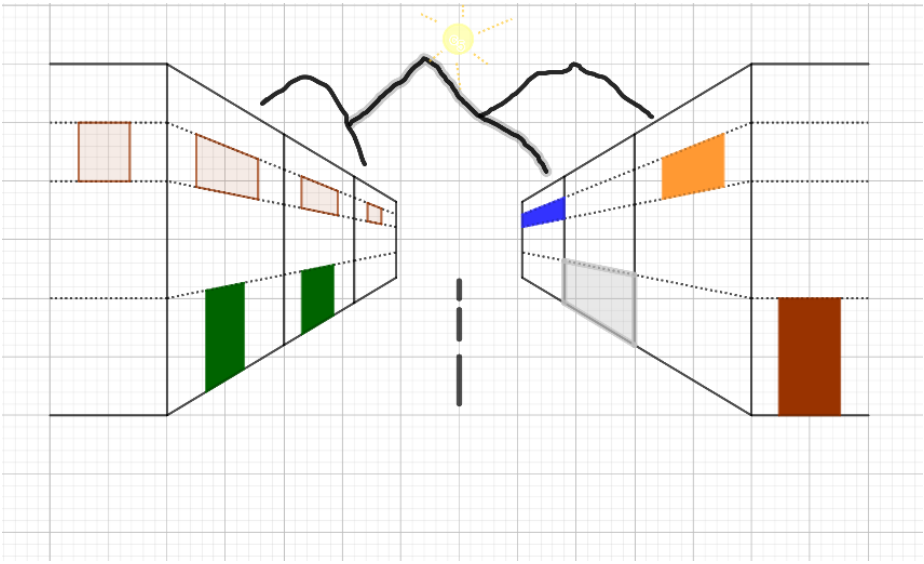
3. Rules of perspective drawing (30 minutes)

The teacher introduces the rules, i.e. the procedure in which the second cube from the previous activity was drawn. Students do the procedure in their notebooks; it is done following the construction steps in GeoGebra files perspective_drawing_cube.ggb (Students are shown the construction in GeoGebra, they draw on their own, step by step.)
Procedure:

1) Draw the front side of a cube, with edge 3 cm long, name the vertex A, B, C, D. Draw square ABCD in the middle or at the left bottom of the page.
2) Draw a line h (called horizon) and mark the vanishing point N (draw a line at least 5 cm above the cube and point N in the middle of the paper).
3) Draw dashed lines AN, BN, CN, DN.
4) On dashed line AN choose a point E.
5) Draw a line parallel to line AB which contains point E and name the intersection of that line and line BN with F.
6) Draw a line parallel to line AD containing point E and name the intersection of that line and line DN with H.
7) Draw a line parallel to line CD containing point H and name the intersection of that line and line CN with G.
8) Draw edges (in color), lengths: $\overline{AE}, \overline{BF}, \overline{CG}, \overline{DH}, \overline{EF}, \overline{FG}, \overline{GH}, \overline{HE}$.

Teacher and students discuss the following questions: Is it important where we place the vanishing point? Will the picture of a cube change if we change the vanishing point?
Teacher asks students to define the perspective point (the point from which we observe a cube)? Does it depend on the vanishing point?

	Students imagine how would a cube look depending on the position of point N. The teacher moves point N in the GeoGebra file (perspective_drawing_cube.ggb) to illustrate the change of presentation of the cube depending on the change of the perspective point of the cube, depending on the change of vanishing point N positioned the position of perspective point. 4. Rule to draw two congruent cubes, one behind another in space (5 minutes) In GeoGebra (perspective_drawing_cubes.ggb), students are presented with two congruent cubes according to the rules of perspective drawing. By moving point I and J (vertex of the front edge of cube IJKLMOPR) to overlap with points A and B (vertex of the front edge of cube ABCDEFGH) one point overlaps with another, to prove that these are congruent cubes. After that, cube IJKLMOPR moves in a way that square IJKL overlaps with the square EFGH (points I and J are overlapped with points F and G). It should be emphasized that in this process the length of edges IJKL decreased. However, the teacher indicates the regularity which can be used to present more complex objects (for ex. two congruent cubes placed one behind another in space) with perspective drawing. With help of GeoGebra (relation between objects) teacher states that lines CG and KP are parallel and emphasizes that this is the rule to use in the following drawings. After the activity, before the follow-up in the next lesson (art lesson, for example) students are assigned a homework task to draw another square, with sides length 2 cm. 5. Draw a street (90 minutes) Students draw a preparation for the street in pencil (according to the rules of perspective drawing) which should look like a picture in pective_drawing_street.ggb. The steps of the construction in GeoGebra file follow the steps of the construction steps that students must take to draw a picture on paper. These steps are presented to students while they create their own constructions. 1) In the very center of A3 paper or a few centimeters above, the vanishing int N is determined. (The center of the paper can be identified as intersection of the diagonals of the rectangle – the paper). 2) Along the left edge of the sheet, 3 cm from the top and bottom edge, students draw a rectangle ABCD with sides lengths of 8 cm (side AB length is 8 cm) and 24 cm. Students draw dashed parallel lines AB that are 8 cm, 16 cm, and 20 cm remote from line AB. These lines mark the space where the entrance door and windows are. At this point, to make sure all students completed the first two steps, students draw and color windows and doors. 3) Students draw a half line starting with vanishing point N and containing point B. Similarly, students draw a half-line starting with vanishing point N, containing point C. 4) Students draw a line, ending points of which are intersections of lines BN and CN with lines parallel to line BC and distant around 6 cm. We mark the intersection of the line and half-line BN with S1 and the intersection of a parallel line and half-line CN with S2. 5) Students draw a line parallel to line CS_1, containing point S_2. Mark the intersection of the line and half line BN with S_3. Draw a line parallel to line BC containing a point S_3. Mark the intersection of that line and half line CN with S_4. Draw the line segment $\overline{S_3S_4}$. 6) Students draw a line parallel to line S_2S_3 containing point S_4. Mark the intersection of the line and half line BN with S_5. Draw a line parallel to line BC containing point $S_{5.\ Mark}$ the intersection of that line and half line CN with S_6.

	Draw the line segment $\overline{S_5S_6}$. 7) By connecting the endpoints of dashed line segments on the front of the building and vanishing point N, on the front sides facing the street, we determine the part to draw doors and windows. 8) Students repeat the procedure and draw, symmetrically, the other part of the street., starting from the right edge of the paper. At the end of the activity, students can draw windows and doors on buildings, they can add trees, streets, people, mountains, etc.  Picture is made in program GeoGebra (perspective_drawing_street_the_end.ggb). 6. Homework (or the following Art class): in order to accentuate the depth of the object, the objects that are near are colored in lighter colors and the object that are far are colored in darker colors. Students are assigned the following task: color the nearest building in yellow, the building next to that one in a darker shade of yellow (this is achieved by adding a bit of black). Students continue the procedure until they color all the buildings. In the end of the activities, students are assigned homework - try to draw their room, their bed, etc. By applying the rules introduced in the activity. A bit of history at the end... The teacher asks how we achieve the illusion of depth on paper. Are the students familiar with the methods? Teacher can show some methods of illustration as they developed through history: • Vertical perspective in Ancient Egypt gets the effect of depth by drawing objects that are far smaller and above. • Reverse perspective from 13th and 14th century – closer objects are smaller • 17th century perspective Which of these leaves the best impression of depth and is that achieved?
	1. Students apply the rules they learned about drawing in GeoGebra and make the materials that teacher uses in teaching 2. Additionally, rules can be introduced for drawing with two or three vanishing points, as well as the situations in which we would use these

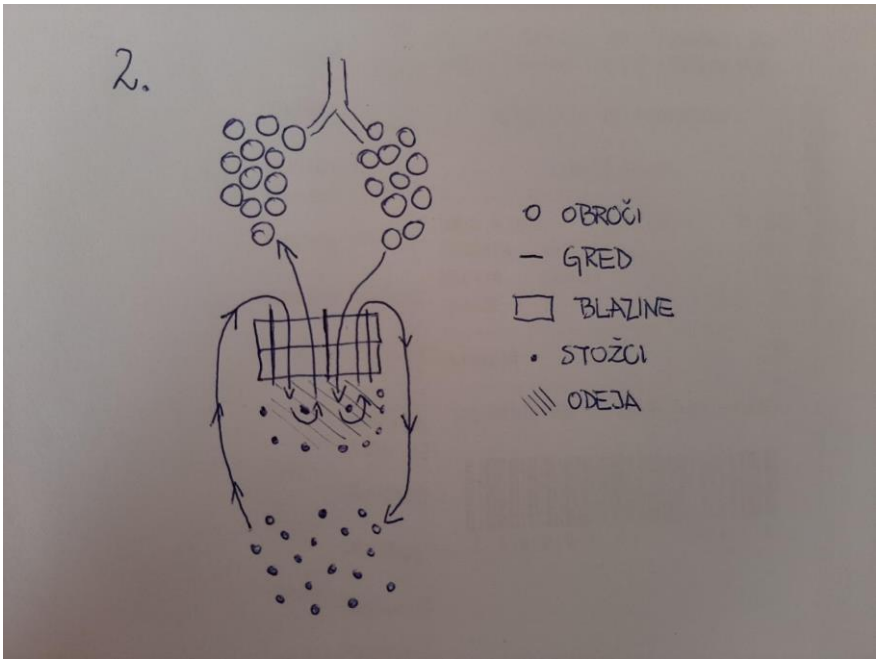
	rules of drawing. By applying these rules, students can draw a cube with two (cube_2points.ggb) and three vanishing points (cube_3points.ggb) in GeoGebra. 3. Teacher can print out the street sketch perspective_drawing_street.ggb)for younger students and they can draw windows, doors, street, trees, people... 4. Teacher can motivate students additionally by organizing a math exhibition at school. More details about math exhibition can be found in the article *How to organize a Math exhibition – Taxi geometry exhibition.* (http://mis.element.hr/list/30/broj/113/clanak/1547/kako-organizirati-matematicku-izlozbu-izlozba-taxi-geometrija).
Additional notes:	
Authors:	Bojan Crnković, Vedrana Mikulić Crnković, Ivona Traunkar (Faculty of Mathematics, University of Rijeka)

Title	**Circulatory system**
Key words	circulatory system, heart, blood, vessels, heart rate
Short description	In this unit students: • learn about the circulatory system (blood, vessels, heart) through physical activity, • measure heart rate and they collect and organise data, • present, read, and interpret the collected data.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: Mother tongue A7: Foreign language Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: collecting and organising data
	A2: circulatory system
	A5: developing motor and functional skills
Expected prior knowledge	
Expected learning outcomes	LEARNING OBJECTIVES – NATURAL SCIENCE: • are able to explain the basic meaning of blood and name blood cells, • can list the components of the heart, • explain that blood flows through the blood vessels, that it is driven by the heart and that the circulatory system is closed, • measure their heart rate, • design and carry out a simple study to find out how the heart rate changes with physical exertion. • know how to show the position of the heart, • are able to name the blood vessels and the differences between them, • know the role of blood in the body and its composition, • name the most common cardiovascular diseases and ways to prevent them, • name the four types of blood groups. LEARNING OBJECTIVES - SPORT: • developing motor and functional skills • developing general aerobic endurance when performing natural forms of movement • performing natural forms of movement (walking on all fours, crawling, single-leg jumps, double-leg jumps) • developing tolerance and positive attitude towards sport • developing the need for daily exercise LEARNING OBJECTIVES – MATHS: • Students solve a problem that requires collecting and organising data, presenting it, and reading and interpreting it.

Expected duration	135 minutes
Preparation	The following should be prepared: PTT, stopwatch, workbook (Appendix), torso model, model of the circulation of blood through the heart, model of the circulation of blood, plasticine blood cells, yellow coloured water - plasma, glass jar, material for the polygon (low cones, masking tape, 15 rings, 4 mats, 3 benches, 20 cones, 50 red papers, 50 blue papers and a sheet).
Detailed description of activities	**INTRODUCTORY PART/WARM-UP** *best done in the gym, but can also be done in the hallway or yard. Distribute the workbook. On the first page there is an outline of the human body (task 1). We instruct them to draw a heart in the outline of the body where they think it is. We then check the sketches together and discuss the position of the heart. Then we explain that the heart is at chest level, in the middle of the body, not on the left side. The heart is about the size of a fist. **MAIN PART** We ask the pupils what the heart's function/role is. The heart is a powerful muscle that works night and day. It pumps blood throughout the body. Blood flows through blood vessels. We show the learners the model of a heart and use it to explain the structure of the heart. Learners do Task 2, in which they have to label and name the parts of the heart (Task 2). Using the model of the body and pulmonary circulation we explain how blood circulates through the body. We also show a video on how blood circulates through the body. We explain the role of blood vessels in the body. A blood vessel is a tubular organ through which blood flows. We describe the structure and function of each blood vessel (arteries, veins and capillaries). They do the third task in the workbook. Have you noticed that blood can be dark red and light red? Why? The dark red blood flows through the veins. This blood is saturated with carbon dioxide and waste substances. The light red blood flows through the arteries, and this blood is saturated with oxygen. Blood is the body fluid that flows through the blood vessels. It carries different substances (oxygen, carbon dioxide, nutrients) through our body to the cells. The human circulatory system is closed, meaning that blood is constantly circulating through the blood vessels in the body. We explain the composition of blood using an experiment (Task 4). We divide the learners into groups of four. Each group receives cups with blood cells (erythrocytes, platelets and leucocytes), which we have moulded out of plasticine, a glass cup and 'plasma' (yellow food colouring mixed with water). We tell the learners to pour "plasma" into a glass jar. We tell them that plasma is a liquid containing red and white blood cells and blood platelets. Then they add to the plasma: - red blood cells or erythrocytes (they supply the body with oxygen), - white blood cells, or leukocytes – they protect the body against infections, bacteria, - blood platelets or platelets - they clog wounds (when we cut or scratch ourselves), preventing large blood losses. They then have a few minutes to observe the composition of the "blood" in the glass jar.

	This is followed by the exercise of measuring the heart rate (Task 5). We explain that one can measure the heartbeat by pressing the index and middle fingers of one hand on the opposite wrist, just below the base of the thumb. Each learner tries to feel their own heartbeat. We ask the learners to start measuring the heart rate on their wrist on cue by counting the number of beats. When we say 'stop', the learners stop counting. They count the beats for 10 seconds. Then they multiply the result by 6 to get the number of beats per minute. They record their heart rate in the table in exercise 5 a). This is followed by an exercise to find out which physical activity results in the highest heart rate. Three exercises are performed in order of difficulty. We start with walking, followed by an easy jog and finally frog jumps. Each activity is performed for exactly 2 minutes. After 2 minutes, each learner measures his/her heart rate. They measure it for 10 seconds and multiply the result by 6 to get the number of beats per minute. After each exercise, there is a short pause to allow the heart rate to return to normal. The results are recorded at exercise 6. b). We discuss the results. The learners record the results in a bar chart. We talk to the learners about blood donation and its importance. We talk about the types of blood groups. There are several types of blood groups. Blood varies from person to person in terms of blood factors (antigens). These are inherited and determine the characteristics of a person's blood. There are 4 basic blood types: A, B, AB and 0. We tell them to do task 6. We tell them to do task 7. Together, we discuss the most common cardiovascular diseases (increased blood pressure, elevated cholesterol levels, heart rhythm disturbances or arrhythmias, heart failure…). We tell them to do task 8. **CONCLUSION:** We talk about what harms our heart. We talk about how to reduce the risk of cardiovascular disease. Using the blood circulation model, we work together to recreate how the blood circulates in humans. The blood coming from the lungs is enriched with oxygen. It enters the heart through the left atrium, then goes into the left ventricle. The left ventricle contracts and pushes the oxygen-rich blood around the body to all the cells. From the cells, blood enriched with carbon dioxide is returned to the heart. It now enters the heart through the right atrium and goes into the right ventricle. The right ventricle pushes blood into the lungs. Here the blood gives up carbon dioxide and takes in oxygen. The cycle is then repeated. We tell them to solve the last task in the workbook (task 9). Final lesson: General warm-up: Elementary game: little atoms One of the learners is designated as the chaser. We put a scarf in his/her hand so that everyone knows who the chaser is. The other learners run around the room freely. We call out a number (e.g. three). The learners should quickly form groups of three. The chaser can no longer catch the groups of three, but he/she can catch all the others. When he/she catches someone, that learner leaves the field and does 5 squats, when he/she is caught a second time, he/she does 5 push-ups, when he/she is caught a third time, he/she does 5 sit-ups, then

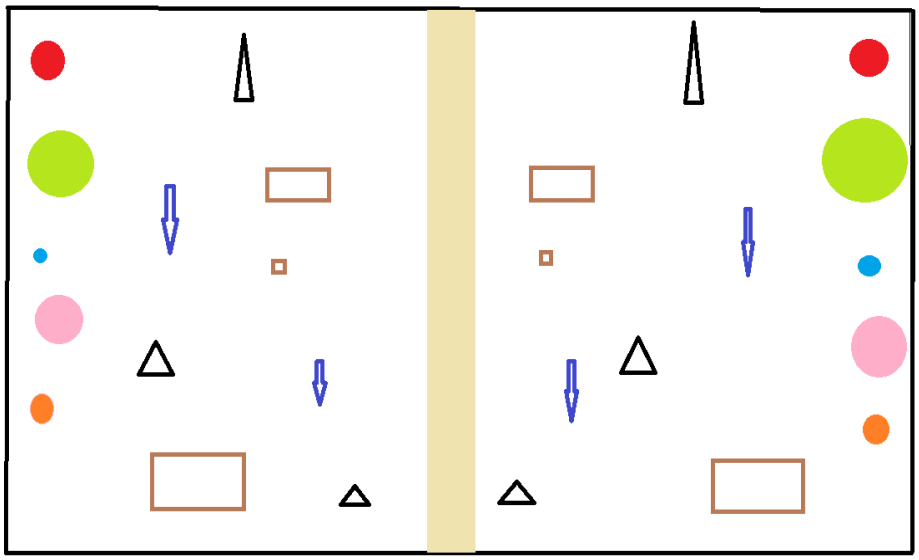
	he/she comes back into the game. After a certain amount of time, we change the chasers. The intensity of the elementary game is increased by adding one more chaser. We limit the game to a smaller space due to the field set up earlier. Elementary game: Running The learners run at a conversational pace. Specific warm-up: We tell the learners to stand in front of me so that they have enough space around them to do the gymnastic exercises (v-shape formation). The teacher shows them various stretching and strengthening gymnastic exercises for all the major muscle groups, which are going to be more stressed during the main part of the lesson. Preparation: The main part of the lesson will be a field. The purpose of the field is to develop general aerobic endurance and to understand how the blood circulates. The exercises the learners will do are: 1. double-leg jumps, 2. running, 3. crawling, 4. one-leg jumps, 5. running, 6. crawling and 7. double-leg jumps. After they complete the field, stretching exercises are performed to stretch the larger muscle groups that were most active during the exercises, to prevent later muscle soreness. After the stretching exercises, the learners lie down on the floor and do relaxation breathing exercises to calm down. Method reparation: Because the field is different from the rest - it shows the blood circulation - it is prepared independently beforehand. We tell the pupils to stand at the first station. We give them instructions on how to do the exercise at the first station and then move on to the second station. We demonstrate each exercise to the pupils. When we say "start", students start the first task on the field. When we say "stop", they finish the exercises. We give the learners the following instructions: 'Imagine you are a red blood cell. Your job is to carry oxygen to all the cells in your body. You start your journey in the left lung wing - here you do double-leg jumps from ring to ring, where you pick up the red card that represents oxygen. You then run from the lungs to the left side of the heart. Here you crawl under the mats representing the inside of the heart, go around the cone and come out of the 'heart' (the ventricle compresses and sends the blood around the body). You run to the cones representing the capillaries. You start hopping on one leg, putting down the red card at one of the cones and picking up the blue one (the blood gives off oxygen and takes in carbon dioxide). You run to the heart, where you crawl under the mats and go around the cone (the ventricle). Once you have crawled out of the 'heart', you run to the right lung wing - here you start to do the double-leg jumps from ring to ring, where you give up the blue card (carbon

	dioxide) and take the red card (oxygen); repeat the blood circulation several times.  At the end of the exercises, we do some stretching exercises: Exercise 1: elbow extensor stretch Exercise 2: Stretching the lateral trunk flexors and the shoulder adductors Exercise 3: knee extensor stretch Exercise 4: hip extensor stretch Exercise 5: stretching the hip flexors and trunk rotators Quantitative and organisational preparation: In the main part of the lesson 3 minutes are devoted to demonstrations of how to perform the exercises on the field. The exercises are first performed for 4 minutes, then there is a one-minute break, then 6 minutes, a minute break, and finally the field exercises are performed for 8 minutes. It will take approximately 20 minutes to complete the field with breaks. During the actual exercises, the teacher walks around the stations and observes the learners to see if they are doing the tasks correctly and if there are any mistakes, he/she corrects them and shows them the correct way to do it. After the exercises, we do the stretching exercises, which takes 4-5 minutes. The last 3 minutes are dedicated to tidying up the field, playing to calm down and relax. Conclusion: A relaxing game. Learners lie down freely in the gym. We guide them through the relaxation breathing exercises. It is important to take deep breaths in through the nose and out through the mouth. They pay attention to calming their heart rate.
Extension activities	Using prepared materials, similar activities can be carried out in higher grades. Similar activities can be carried out with other topics of the human body.
Additional notes	adapted from the master's thesis by Veronika Valič
	Appendix 1: Workbook
Authors	Marina Volk, Tadeja Volmut, Nataša Dolenc Orbanić

Title	**Survey**
Key words	organize, present, and interpret data, substances, properties, sports, sports equipment
Short description	Students plan an investigation into the characteristics of balls they know, e.g., handball, basketball, volleyball, football ball, tennis ball, ping-pong ball, sponge ball, etc. Students plan an investigation into how balls differ from each other - what tools they would need to find out the differences, what investigations they would conduct, how they would record the data, etc. Students sort, classify, and rank balls according to their properties (e.g., kneadability, compressibility, hardness, mass) and explain how the properties of substances relate to their uses.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: **PE** A6: Mother tongue A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: collect, organise, present, and interpret data
	A2: Different substances
	A5: Sports equipment
	A7: sports, sports equipment
Expected prior knowledge	substances properties (kneadability, compressibility, hardness, density)
Expected learning outcomes	MATHEMATICS: - to record the count in a spreadsheet; - to collect, organise, present and interpret data. NATURAL SCIENCE: - to classify, rank and order substances according to their properties (kneadability, compressibility, hardness, density); - explain how the properties of substances are related to their use. PHYSICAL EDUCATION - to know how to handle different sports equipment (different types of balls); - to name sports equipment and know how to use them safely. ENGLISH: - to describe different sports and sports equipment - to identify different types of sports equipment (balls) and relate them to the corresponding sports - to use a comparative in English by comparing balls to each other (in size, weight, etc.: bigger, smaller, heavier, lighter)
Expected duration	135 minutes
Preparation	TEACHING RESOURCES AND MATERIALS: Appendix 1: SURVEY DESIGN Appendix 2: learning sheets at stations Appendix 3: BINGO Appendix 4: TV guide worksheet Appendix 5: personal sports chart Differents balls: a soft cloth ball, a lightweight children's ball, a football, a

	handball, a medicine ball and a table tennis ball …
Detailed description of activities	**INTRODUCTORY SECTION** The introductory lesson is conducted in English, where students use different balls to describe the sports in which the ball is used, describing the balls. The teacher brings a variety of balls into the classroom (a soft cloth ball, a lightweight children's ball, a football, a handball, a medicine ball and a table tennis ball) and discusses with the pupils what the balls are made of and what they are used for. The balls can be brought in a large bag from which the pupils draw the balls, or the pupils can close their eyes and pat the ball. The teacher asks them, for example: - Is the ball soft or hard?/Can you squeeze the ball?/Is it big or small? In which sport do you think we use it? Together they describe all the balls and their properties, and also learn to name the material each ball is made of (helpful: https://discover.hubpages.com/games-hobbies/Different-types-of-balls-and-their-specialties). Students also test how much the balls bounce, how much they can squeeze, how heavy they are, etc. and compare them (e.g. A tennis ball is smaller/lighter than a football.) In the second part, pupils work in pairs or groups to write a "guidebook" for different sports (e.g. write the name of the sport, the number of players and the equipment/props). They focus on the description of the aids.) **MAIN PART OF THE LESSON** Divide the pupils into groups of 4: **STEP ONE** - Planning the research - Conduct a discussion with the students: - Who is the researcher? The person who is doing the research. - What qualities does a researcher need to have? He or she must be meticulous, cooperative, flexible, persistent, attentive, insightful, resourceful, ... - What is important before starting research? Research design: purpose, method, tools, anticipation, summarising final findings. - Look around. Can you deduce what the topic of our research will be? Balls, characteristics. We can have a conversation with the pupils, or we can drop the conversation and ask them an open question: In what ways would you find out how balls differ from each other, what tools would you need to find out the differences between balls? Guided discussion before the open question: - In which aggregate state are the balls? Solid aggregate state. - What are solids? Substances that hold and maintain a particular shape. - What characterises a solid aggregate? Objects do not move when placed, e.g. a chess piece. The volume/mass does not change although the shape changes, e.g. an inflatable ball. The volume/mass changes when an external force is applied to it, e.g. heating, forging, cutting, squeezing, grinding, kneading, breaking, crushing, etc. - What properties are observed for solids in natural science and have you observed them in english? Hardness, flexibility, compressibility, permeability to water and air, size, colour, substance. - What can a substance be in terms of hardness? Soft or hard. - How do we determine hardness? We compare two objects to each other by rubbing or feeling. - What does flexibility mean? An object can be bent, stretched, squeezed and

	it will return to its original shape. - What can a substance be in terms of flexibility? Flexible: the shape does not change because it returns to its original shape; inflexible: if the object's shape changes. - What substances can solids be made of? Plastics, cloth, water, metal, rubber ... - What do you know about the mass of solids? We measure with a balance. We express it in numbers and units of mass: t, kg, dag, g and mg. Cutting or taking away a solid change the mass. - What else can we measure? How do we record it? Size, with numbers and units of length. What are some ways to find out how the balls differ from each other, what tools would you need to find out the differences between the balls? Divide the pupils into groups of 3. Give each group of 3 a question sheet (Annex 1). They have five minutes to plan their investigation. They write their answers on the sheet. - What are the differences between the balls? Colour, material, hardness, weight, compressibility - flexibility, usability. - In what ways would you find out how balls differ from each other? By weighing, touching, bouncing and measuring. - What tools would you need to find out the differences between balls? A tape measure, a balance, a ruler, a string. Sharing ideas Students write down and then report on how they planned the research in the group (so the teacher can correct or complete any wrong plans). **STEP TWO** - Carrying out the research. Students will investigate the properties of the balls at the different stations and record their findings and comparisons (each station should have at least 4 different balls). Presentation of the stations: Station 1 STICKING: it belongs to flexibility. If you squeeze an object, it returns to its original shape. Station 2: you will arrange the balls in order of hardness from the softest to the hardest. You will be assessed by touch and force - squeezing. Station 3: you will measure the mass of the balls using a balance. Pay attention to the correct setting of the scales: unit of measurement gram and start at zero grams. Station 4: You will measure the height of the bounce (e.g. handball, medicine ball, tennis ball, table tennis ball, cloth ball). A pupil drops the ball from a certain height, another marks the height of the bounce on a tape measure, a third writes the figure on a worksheet. Station 5 SIZE: using a string and a tape measure, you will measure the size of an object around the outside (its circumference). Station 7: you will find out what substance the ball is made of. 8. The FLOATING station: you will find out whether the ball floats or sinks. **STEP THREE** - Reporting Students report their findings, and the teacher may ask them sub-questions: - What are the characteristics of the balls that make them different? - Which ball bounced the highest? What influences this? - Why are some balls easier to guide than others? - Which ball is easier to catch and why? - Which ball has the largest mass and which has the smallest mass? Does the

	mass of a ball affect its usability? If you needed a ball that you had to roll 50 metres with your hand, which one would you choose? Justify your answer. **CONCLUSION** Elementary game with different balls Space: gym or outdoor playground Equipment: 8 to 10 different balls (no medicine balls!); bats, cones of different sizes, boxes of different sizes, yoghurt pots, etc. Source: https://www.playpartyplan.com/ball-games/ Pupils are divided into two groups of equal numbers. Each group is given the same number of balls, which are placed on the edge of the gym. In the middle of the field, benches (or a low net - no more than waist-high for the children) are placed to divide the field into two halves. The first group of pupils is placed on one half of the field and the second group on the other. In both fields, the teacher, with the help of the pupils, symmetrically places the different utensils (sticks, different cones, yoghurt pots, boxes of different sizes, etc.). On the teacher's signal, the pupils try to compact the placed utensils. The pupils are asked to knock down the objects according to the content of their sport lesson (example: from in front of the chest, from above the head, etc. ; with an elbow pass (right and left hand), a lower or an upper bounce). The first group to knock down all the implements placed in the opposite group wins the game. 

	- bench  - balls  - boxes  - kij  - cones
Extension activities	Using prepared materials, similar activities can be conducted in higher grades with more complex survey.
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanič, Mojca Žefran, Tadeja Volmut (Department of Elementary school, University of Primorska)

Title	**The Fibonacci sequence**
Key words	The Fibonacci sequence, the Fibonacci numbers in nature
Short description	This activity develops pupils' calculation and application skills and inspires them to create their own circle artwork. In this unit students: •learn about and continue the Fibonacci sequence, • learn how the Fibonacci sequence occurs in nature, •create a sequence. Students in 4th grade learn about Fibonacci and the Fibonacci sequence in English.
IT tools	
Fields (select)	A1: **Maths** A2: **Natural sciences** A3: Art A4: Music A5: PE A6: Mother tongue A7: **Foreign language** Other: A8: IT A9: Citizenship
Themes (for each selected field)	A1: the Fibonacci sequence, creating a sequence
	A2: the Fibonacci sequence in nature
	A7: calculating and describing the Fibonacci sequence in English
Expected prior knowledge	calculating the sequence
Expected learning outcomes	MATHEMATICS • learns about and continues the Fibonacci sequence • creates a sequence, NATURAL SCIENCE • learns how the Fibonacci sequence occurs in nature, ENGLISH • calculates the numbers in the Fibonacci sequence in English, • recognises the occurrence of Fibonacci numbers in nature based on descriptions in English, • learns the English terms such as "sequence, spiral, clockwise, counter-clockwise, pinecone, shell, pineapple, daisy..."
Expected duration	90 minutes
Preparation	The following should be prepared (see appendices): drawing grid, examples of the Fibonacci sequence from nature, Fibonacci colouring sheets
Detailed description of activities	**THE ENGLISH LESSON** 11. WARM-UP In the introduction, pupils learn about Fibonacci and the Fibonacci sequence in English. The teacher can show them the following video: https://www.youtube.com/watch?v=ihxJN6ZC9HE or he/she can introduce the Fibonacci sequence in a similar way on his/her own. Together, they review English expressions for addition (*plus, equals*) and work out how to calculate the numbers in the Fibonacci sequence. **12. MAIN PART** 1. Calculating the successive numbers in the Fibonacci sequence: learners try to calculate a few more numbers in the sequence (they should continue beyond 100

and name the numbers (in English); e.g. "fifty-five plus eighty-nine equals one hundred and forty-four"). When they have written the numbers down, the teacher says: *This is the Fibonacci sequence.*

2. Fibonacci numbers in nature: the teacher shows the learners some pictures (he/she can also bring a cone or a seashell) where they can identify the numbers in the Fibonacci sequence (pinecone, pineapple, shell, daisy, cauliflower, sunflower...).

What do you see? A spiral. Can you find more spirals?

3. The Fibonacci hunt: the teacher makes descriptions of things that show the Fibonacci sequence and the learners guess (first they can do this orally, then they get a worksheet where they read the descriptions and guess). Example of a description: *Its scales are arranged in a spiral. It is brown on the outside and yellow on the inside. It is a tropical fruit.*

(for a more challenging game, you can e.g. include different plants and flowers with different numbers of petals).

3. CONCLUSION

In the final part, learners can work in pairs to play "the Fibonacci hunt" or work together to find another example of the Fibonacci sequence in nature.

AFTER THE ENGLISH LESSON – THE MAIN LESSON

1. INTRODUCTORY PART

Together with the learners we revise the numbers in the Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, and invite the learners to calculate a few more. Tell them that the numbers in this sequence are infinite, because we can always add the two adjacent numbers to get a new number.

We introduce who Fibonacci was briefly:

*Fibonacci was born around 1170 to an Italian merchant. He travelled around the world a lot with his father and was educated in Algeria, where he learned about the Hindu-Arabic numeral system (**today's Arabic numerals**), which he took to Europe. There he wrote the Book of the Abacus, where he described the advantages of these numerals compared to the Roman numerals. In Italy he became the pre-eminent mathematician of his time, dying in Pisa around 1240.*

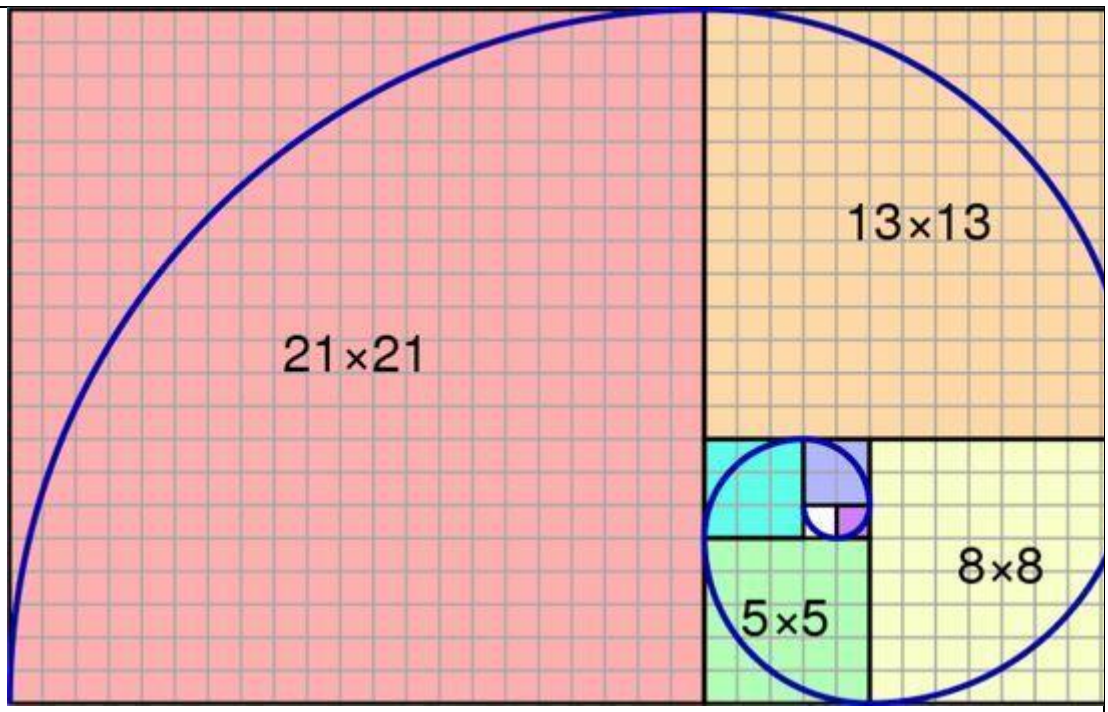
With the learners we revise the content from the English lesson: where the F. sequence occurs in nature or where we find numbers that are part of the Fibonacci sequence (the arrangement of petals in flowers, the arrangement of scales in pineapples, etc.).

2. MAIN PART

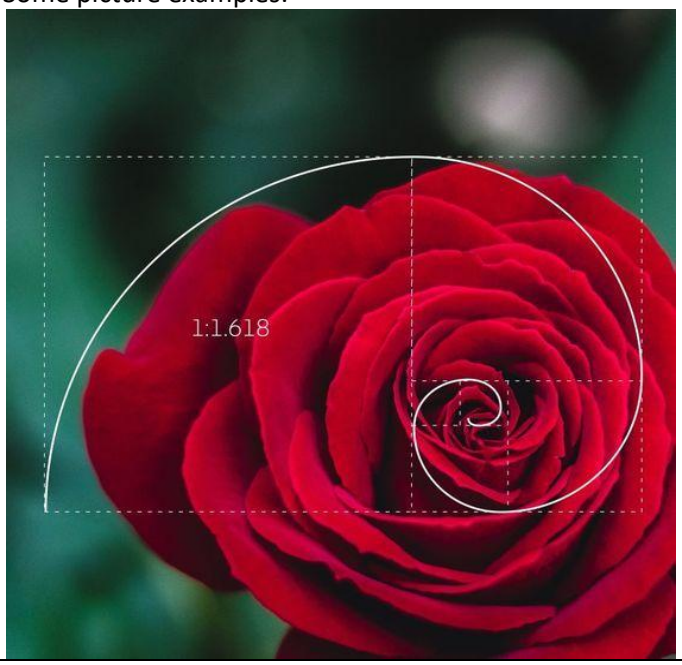
Colouring the grid and drawing the spiral

The learners are given a square grid in which they colour the number of squares according to the F. sequence (starting approximately in the bottom left, they colour each number in the sequence with a different colour). When they have painted up to the number 13 or 21, they try to draw a spiral. The teacher guides them through the colouring.

Example:



We ask the learners what the spiral they got reminds them of - a seashell, a spiral staircase, a rose flower, etc.
Some picture examples:



	  To relax learners can be given a Fibonacci spiral to colour (one example below). Pupils find the beginning of the spiral and colour the picture as they wish. **3. CONCLUSION:** We revise how the Fibonacci sequence is formed (we get the new number by adding the previous two numbers). Then ask the learners to think of a sequence of numbers, but without telling the rule of the sequence. Some sequences are written on the board and the learners try to work out the rule of the sequence.
Extension activities	Some useful materials for the English lesson: Activities: https://www.mensaforkids.org/teach/lesson-plans/fabulous-fibonacci/ https://blog.doublehelix.csiro.au/fibonacci-fruit/#:~:text=The%20sequence%20goes%3A%201%2C%201,12%20scales%20on%20your%20pineapple. Fibonacci flowers: https://www.pansymaiden. com/flowers/types/fibonacci-flowers/
Additional notes	
Authors	Marina Volk, Nataša Dolenc Orbanić, Tadeja Volmut, Mojca Žefran (University of Primorska, Faculty of Education)

Title	**Timeline**
Keywords	Comparing numbers, tens, hundreds, thousands
Short description	As part of the activity, students will get to know the definitions of decades, centuries and millennia, through learning tens, hundreds and thousands
ICT tools included	Smart bord, PowerPoint
Areas (select)	**A1: Mathematics** **A2: Natural science** A3: Art culture A4: Musical culture A5: Physical culture A6: Mother tongue A7: Foreign language Other: **A8: Informatics** A9: Civic education
Topics (for each selected area)	A1: number line
	A2: timeline
	A8: creation a timeline using ICT tools
Expected prior knowledge of students	Comparing the number of concrete objects in sets ("less", "more", "equal", "one more", "one less")
Expected outcomes	Differentiating even and odd numbers, and noticing number patterns, eg predecessors and successors Using ordinal numbers up to 1000 Determining the course of time and time intervals in everyday situations
Expected duration of the activity	90 minutes
Preparation of activities	1. Prepare an analog and digital clock to compare shapes, ways of writing numbers, Arabic and Roman numerals 2. Blank A4 papers for assignments 4. Work material with tasks
Detailed description of all teaching activities	In the introductory part of the class, the teacher shows the students what an analog clock looks like and what a digital clock looks like and what the differences are in shape, the way of looking at the clock, the way of writing and reading numbers (eg 6 in the morning is 6 in the afternoon - how and why?). On blank papers, students should write the default Arabic words in Roman numerals. It shows that the day as a whole has its parts. In the second part of the activity, the students repeat what a month is, how many months there are in a year, what months they are, what are the ordinal numbers for each month, and how many days each month has. which month has more or less days, why does february have 28(29) days. In this way, students compare numbers (>, <, =) Students are introduced to the method of determining the number (counting) of the days of the month using the joints of the fingers.

	In the third part of the activity, students are introduced to the definitions of decade, century and millennium. According to the example on the smart board that the teacher will draw, the students should make a timeline of a century using tens on the number line in PowerPoint (SmartArt), changing of color, style, size. It is explained to the students that there is a decade system in mathematics, where everything is calculated using the number 10. Next, students should determine the number of years in a decade, a century and a millennium. 10 years is - 1 decade 100 years is - 10 decades or 1 century 1000 years is - 100 decades or 10 centuries or 1 millennium In the final part of the activity, students should 1. write years in words and determine how many decades, centuries and millennia there are in that year 200 987 109 1099 456 in 2021 2. list all the years (ordinal numbers) between 998 and 1012. Which of these years belong to which millennium?
Possibilities to expand activities	It is possible to prepare similar activities on the computer, Creation of timelines in Timeline Knightlab
Additional notes	
Authors	Antea Čilić, Hrvoje Ljubić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Roman day –numbers up to 20**
Keywords	Roman numerals, time
Short description	As part of this activity, the students will repeat the Roman numerals through the exercise, one of the students throws a dice, and the result should be calculated, the result represents the number of repetitions of the exercise. Furthermore, students should recognize the Arabic numerals as belonging to the Roman numerals. With the help of a memory game, students compete in groups to find out more pairs of numbers in the combination of an Arabic number with its Roman notation in as few moves as possible. Using a clock with Roman numerals, students learn that time, hours, years and dates are most often recorded with Roman numerals.
ICT tools included	Computer, projector 3D printer
Areas (select)	**A1: Mathematics** **A2: Natural science** A3: **Art culture** A4: Musical culture A5: **Physical culture** A6: Mather tongue A7: Foreign language Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: Roman numerals
	A2: time
	A3: drawing, cutting, coloring
	A5: squats, skipping rope
Expected prior knowledge of students	Numbers up to 20, counting time
Expected outcomes	Knowledge and calculation of Roman numerals
Expected duration of the activity	90 minutes
Preparation of activities	Instruct students to bring prepared cubes made of paper or styrofoam from home Write the numbers I-VI on the cubes Prepare chart paper for first activity Make a memory game using paper or open an online available game on InaMath online coruse in Special Days if a projector and a computer are available. Prepare the clock mechanism and the plate to which the mechanism can be connected. Students should have tempera or acrylic paints.
Detailed description of all teaching activities	In the first part of the activity through an exercise, students repeat Roman numerals, one of the students throws dice (with Roman numerals and operations), and they need to calculate what the result is, the result represents the number of repetitions of an exercise

	1.number	operation	2. number	repetition
squats				
Jumping rope				

In the second part, students play the Memory game.
Students can compete in groups, which group will reveal more pairs of numbers in as few moves as possible.
Example:
Match the Arabic number with its Roman notation:

7	9
V	VII
5	10
IX	X

The activity can be adapted to the students' knowledge, and much larger numbers can be displayed.

3rd part of the workshop
Prepare the clock mechanism and the plate to which the mechanism can be connected. Students should have tempera or acrylic paints.

Students decides in wich color will paint mechanism and face of the clock.

Then the teacher discusses with the students which Roman numeral should be written where in the lesson.

After everything is dry and the clock is mounted, than students repeat how to read the time.

Possibilities to expand activities	If the school has a 3D printer, it is possible to print a 10-sided body and write the Roman numerals I-X on it as in the picture Download the preparation or create a cube on which the operations will be written (+,-,*,/,+,-) It is also possible to download the microbit code for extracting random Roman numerals and operations
Additional notes	
Authors	Antea Čilić, Hrvoje Ljubić, Mila Zovko (FPMOZ, University of Mostar)

<table>
<tr><td>Title</td><td>Roman day –numbers up to 100</td></tr>
<tr><td>Keywords</td><td>Roman numerals, time</td></tr>
<tr><td>Short description</td><td>As part of this activity, the students will repeat the Roman numerals through the exercise, one of the students throws a dice, and the result should be calculated, the result represents the number of repetitions of the exercise.
Furthermore, students should recognize the Arabic numerals as belonging to the Roman numerals. With the help of a memory game, students compete in groups to find out more pairs of numbers in the combination of an Arabic number with its Roman notation in as few moves as possible.
Using a clock with Roman numerals, students learn that time, hours, years and dates are most often recorded with Roman numerals.</td></tr>
<tr><td>ICT tools included</td><td>Computer, projector
3D printer</td></tr>
<tr><td>Areas (select)</td><td>A1: Mathematics
A2: Natural science
A3: Art culture
A4: Musical culture
A5: Physical culture
A6: Mather tongue
A7: Foreign language
Other:
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<tr><td>A2: time</td></tr>
<tr><td>A6: grammatic</td></tr>
<tr><td>A5: squats, skipping rope</td></tr>
<tr><td>Expected prior knowledge of students</td><td>Numbers up to 100, counting time</td></tr>
<tr><td>Expected outcomes</td><td>Knowledge and calculation of Roman numerals</td></tr>
<tr><td>Expected duration of the activity</td><td>90 minutes</td></tr>
<tr><td>Preparation of activities</td><td>Instruct students to bring prepared cubes made of paper or styrofoam from home
Write the numbers I-VI on the cubes
Prepare chart paper for first activity
Make a memory game using paper or open an online available game on InaMath online coruse in Special Days if a projector and a computer are available.</td></tr>
<tr><td>Detailed description of all teaching activities</td><td>In the first part of the activity through an exercise, students repeat Roman numerals, one of the students throws dice (with Roman numerals and operations), and they need to calculate what the result is, the result represents the number of repetitions of an exercise
<table>
<tr><td></td><td>1.number</td><td>operation</td><td>2. number</td><td>repetition</td></tr>
<tr><td>squats</td><td></td><td></td><td></td><td></td></tr>
<tr><td>Jumping rope</td><td></td><td></td><td></td><td></td></tr>
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For the 2nd part (Croatian language, science, questions can be asked depending on the current material)
Give the students a worksheet on which they will have to choose the correct answer, the answers will be marked with Roman numerals
Activity:
Students answer the questions, at the end it is necessary
write down the Arabic number make up the written Roman numerals in order with the correct answers from the 1st question to the last.

Example of a worksheet:

How many vowels does the Croatian language have?
And the Croatian language has 25 vowels
L The Croatian language has 4 vowels
X The Croatian language has 5 vowels
C The Croatian language has 30 vowels

What date does spring start?
And the first day of spring is March 23
L The first day of spring is June 25
X The first day of spring is April 1
C The first day of spring is March 20

Water that flows over the surface of the land is called:
I Liquid water
L Water land
X Water is missing
C Stagnant water

What is the definition of nouns?
I Nouns are words that denote the names of beings, things and phenomena
L Nouns are words that denote the actions of beings, things and phenomena
X Nouns are words that denote the appearance of beings, things and phenomena
C Nouns are words that denote the state of being, things and phenomena

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In the above case, for example, Roman XCII, Arabic 92

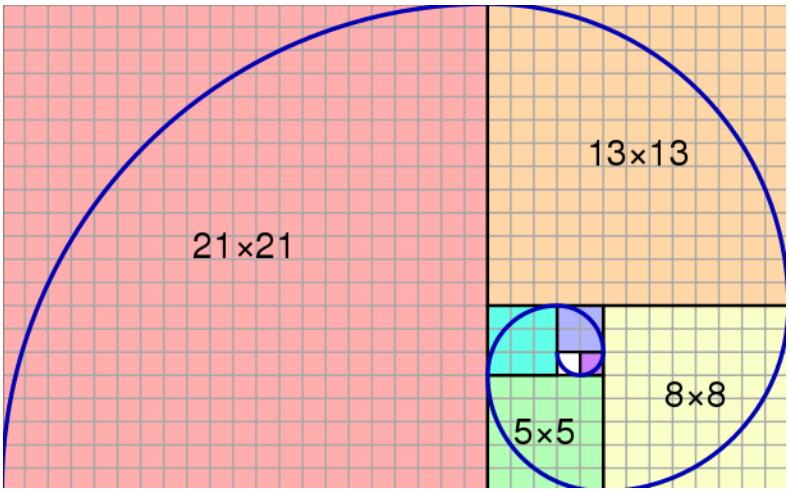
In the 3rd part, students play the Memory game.
Students can compete in groups, which group will reveal more pairs of numbers in as few moves as possible.
Example:
Match the Arabic number with its Roman notation:

7	9	X	IX
V	VII	54	XCII
5	10	C	LIV
8	VIII	92	100

	The activity can be adapted to the students' knowledge, and much larger numbers can be displayed. In the second part, the teacher realizes the display of time on the clock, the conversion of minutes into hours. Bring a watch on which the hours are marked with Roman numerals. Explain to students that Roman numerals are most often used to record time, hours, years, dates... Ask them a mathematical problem, where they have to show the final solution on a clock with Roman numerals. e.g. Marina went to school at 8:00 a.m., she was there for 180 minutes, she walked home for 20 minutes... show on the clock with Roman numerals when she arrived home
Possibilities to expand activities	If the school has a 3D printer, it is possible to print a 10-sided body and write the Roman numerals I-X on it as in the picture Download the preparation or create a cube on which the operations will be written (+,-,*,/,+,-) It is also possible to download the microbit code for extracting random Roman numerals and operations
Additional notes	
Authors	Antea Čilić, Hrvoje Ljubić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Fibonacci numbers**
Keywords	A sequence, a Fibonacci sequence and a spiral
Short description	The aim of the activity is for students to become familiar with the concept of sequence, Fibonacci sequence and spiral
ICT tools included	A4 paper, crayons, geometry accessories, pictures of Fibonacci-like spirals in nature
Areas (select)	**A1: Mathematics** **A2: Natural science** **A3: Art culture** A4: Musical culture A5: Physical culture A6: Mother tongue A7: Foreign language Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: units of measurement for mass (kg, gr..), addition, subtraction, division, sets, ratios (larger/smaller, faster/slower)
	A2: Plant and animal life
	A3: Art and interpretation of the world/Art and science (3.r)
Expected prior knowledge of students	Students know numbers, they know basic mathematical operations with numbers up to 1000
Expected outcomes	Students will be able to explain the concept of sequence, and give some examples of sequnce Students will know how the Fibonacci sequence is formed and the connection with the Fibonacci spiral
Expected duration of the activity	90 minutes
Preparation of activities	Preparation: Prepare A4 papers, "cubes" should be printed on the papers like in the calculation notebook A ruler and a triangle for the board should be prepared Students should have a ruler and a triangle, as well as crayons and writing utensils Pictures of examples of the Fibonacci spiral in nature are prepared
Detailed description of all teaching activities	Activities: PART 1 (15 min) The concept of sequence is explained to the students through examples from real life: 1st example

	We string beads of different colors next to each other on a thread in order to get a bracelet, it is possible to tell at any time which bead is in which place, for example, the first place is a red bead, the second one is blue, then pink, then red again, etc. . 2. example Students stand in line and wait to get on the bus, it is known exactly which student is first in line to get on the bus, which is second in line, etc. The goal is to bring the students to the intuitive conclusion that the order of the members in a sequence is always known, that is, in which place each member of the sequence is located. PART 2- 20 minutes We introduce the students Fibonacci sequence as a sequence of numbers, where the rule of how to calculate in which place which number appears is known There are number one in the first two places, and then each subsequent member is obtained as the sum of the two previous numbers in the sequence. A discussion is started with the students about which numbers we would get in that sequence of numbers from 0 to 100 PART 3-20 minutes Let's explain to the students that the Fibonacci series is connected to the Fibonacci spiral, which we form using the Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21... It is formed in the following way: at the beginning, we place two squares of the side lenght 1 next to each other , which represent the first two Fibonacci numbers. Next to them, place the square of side lenght 2, then the square of side length 3, which rests on the square of side lenght 2 and the square of side lenght 1. Next comes the square of side lenght 5, which rests on the squares of side lenght 2 and 3, and so on. The external edge points at the junction of the newly added square with the square of the previous size form the points through which the Fibonacci spiral passes.



* photo from http://johnshortt.org/nature-by-numbers-part-two/

Discuss with the students where they have seen such a spiral in nature (a snail's house, a seahorse's tail...)

PART 4

Students sketch a Fib.spiral according to the given rules, and they are shown how they can use the spiral to draw a parrot.

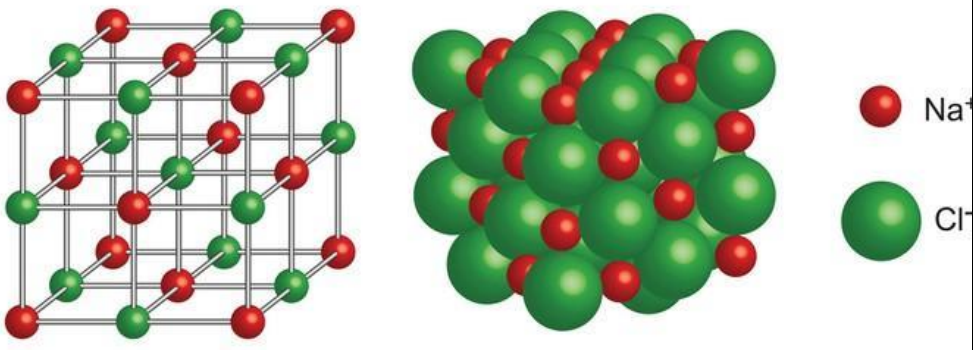
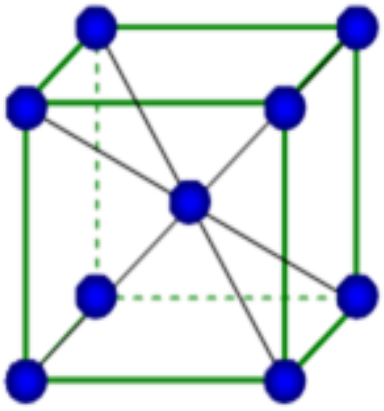
At the end, repeat with the students what a sequence is and what are examples of a sequence in life, and how we get to the Fibonacci sequenceand where we see such spirals in nature.

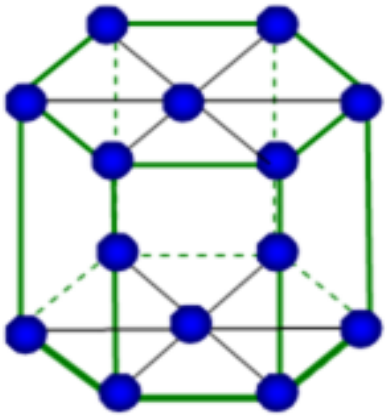
Possibilities to expand activities	
Additional notes	
Authors	Antea Čilić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Little gardeners**
Keywords	Units of measurement for length, plants, parts of a plant, tiling of a plane
Short description	Through two activities, students will have the opportunity to learn about the development of a plant from germination to flowering, through this, in addition to monitoring plant growth, they will learn to use a ruler and read measurement units for length, and compare and transfer measurements from one unit to another
ICT tools included	Ruler, slips of paper for writing a name, for planting :hummus, seeds, cups/pots
Areas (select)	**A1: Mathematics** **A2: Natural science** A3: Art culture A4: Musical culture A5: Physical culture A6: Mother tongue A7: Foreign language Other: A8: Informatics A9: Civic education
Topics (for each selected area)	A1: Length measurement, measurement units A2: Plant and animal life (benefits from plant and animal life)
Expected prior knowledge of students	Students can perform four basic operations with numbers up to 1000 They know the basic measurement units for length
Expected outcomes	The student explains and connects the living conditions and diversity of living beings in different habitats and describes cycles in nature On the example of the plant he grows, he can observe the life cycle of the plant from seed to plant and the way in which the seed is regenerated With guidance, the student explains the results of his own nature research The student uses measurement units for length in everyday life
Expected duration of the activity	90 minutes
Preparation of activities	Two phases First: Prepare pots in which to plant beans or sunflowerseeds, hummus, seeds and slips of paper on which it will be possible to write the student's name Second : Prepare a ruler with a clearly marked scale
Detailed description of all teaching activities	Activities: 1st phase (part of the lesson, 15 minutes) Explain to the students that we will plan and plant a garden in the school yard, and that we need to grow sunflower seedlings. Students are divided into groups of 2 to 3 students and each group should plant its own sunflower and take care of it for the next month.

	It is also possible for each student to bring a sunflower from home after a month if it is not convenient for them to be at school all the time. 2nd phase(75 minutes) After a month, the students are introduced to the scale on the school ruler, the relationship dm/cm/mm is explained (cm is 10 mm, dm is 10 cm, so dm is 10*10mm=100mm), and the height of the sunflower is measured. Then is performed analysis whose sunflower is the tallest, whose is the smallest, and analyzes why this might be so(how much was irrigated the sunflower, was it in a sunny place...). After this activity, plant sunflowers in the school park. Plan what size garden is needed, students need calculate how much space each flower requires, for example, we can say that each sunflower needs a square with a side length of 20 cm of free space, and how we can fit these needs into the existing available space (tilling plane with squares...) It is also possible to make an irrigation plan, e.g That the sunflower is watered every other day, that watering starts with 1 dl of water, and that every week the amount of water for irrigating an individual sunflower is increased by 0.5 dl while the sunflower is in the pot, and that after transplanting them in yard the amount of water per individual sunflower increase by 3 dl of water the first month, by 5 dl of water the second month and that later it is not necessary to increase the amount of water. In this way, it is possible to monitor and ask the students to calculate how much water is needed for each watering during growth in the pot and outside. 1dl->1.5dl->2dl->2.5dl while in the container 2.5dl last week in the container->5.5dl during the first month outside-> 10.5dl during the following months
Possibilities to expand activities	If it is not in season, beans or some other suitable plant can be planted The concept of average value and how to calculate the average height of a sunflower can also be explained. It is also possible to calculate the total amount of irrigation and the total water consumption during one watering, during the week, during the month..
Additional notes	
Authors	Antea Čilić, Mila Zovko (FPMOZ, University of Mostar)

Title	**Atomic mathematics**
Key words	Geometric shapes such as hexagonal prisms, cube, cuboid, etc. Atom Chemical bond Crystal lattice
Summary	This scenario connects the concept of atom, chemical bond and crystal lattice with lessons from mathematics related to geometric shapes. Pupils will have the opportunity to recognize the correspondence between the shapes of crystal lattices of different molecules and geometric shapes. In addition, pupils will be given the task to draw or make selected crystal lattices from plasticine and toothpicks.
ICT tools included in scenario	*Drawing program (Microsoft Paint)
Areas	A1: Mathematics A2: Science A3*: Arts A4*: Informatics
Topics (for each selected Area)	A1: Geometric shapes
	A2: Atoms, chemical bonds and crystal lattices
	A3*: Practicing different techniques of drawing with wooden crayons/crayons/watercolours or working with plasticine.
	A4*: Drawing on computer using an available drawing program (e.g. Microsoft Paint)
Expected prior knowledge of pupils	Recognition of geometric shapes such as hexagonal prism, cube, cuboid, etc. *Basic knowledge regarding drawing techniques and working with plasticine *Basic knowledge regarding working on computer
Expected outcomes	Established knowledge about the appearance of different geometric shapes; Introduction to the concept of atom, chemical bond and crystal lattice; Introduction to the fact that chemical bonds are often arranged in space like geometric bodies - mathematics is all around us!
Expected duration of the activity	2 school hours (90 mins in total), with the possibility of extension to additional art and computer science classes
Preparation activities	Additional material for implementation of this scenario please download from the link - https://inamath.uniri.hr/wp-content/uploads/2022/12/Atomic-Mathematics.pptx
Detailed description of activities	Activity 1: Introducing pupils to the concept of atoms and chemical bonds Start with the simple explanation that atoms are the smallest particles from which everything in the world is built. There are 118 different types of atoms in nature (which are all arranged in the Periodic table of elements) and everything in the world is built from various combinations of those 118 atoms. Atoms are connected to each other by chemical bonds that hold them together (like ropes) and thus form molecules and crystal lattices. Complete the explanation with a question: if we cut/tear up the paper, to how small parts can we cut/tear up it? Answer is: to the size of atoms which make paper. Activity 2: Repeating what types of geometric shapes exist The teacher repeats with the pupils how a hexagonal prism, cube, cuboid etc. looks like. Activity 3: Relating the appearance of chemical lattices to geometric solids

	The teacher presents different crystal lattices to the pupils and gives them the task to connect them with corresponding geometric shapes. It should be pointed out to the pupils that an object which is made of a crystal lattice, e.g. salt, an object made of iron (nail), an object made of zinc (nail), graphite (graphite pen), etc., are in fact a combination of a large number of crystal lattice units, as seen in the examples below. Crystal lattices cannot be seen with the naked eye, but if we look at these objects through powerful microscopes, we would see them. Example 1: Crystal lattice of salt Salt consists of sodium and chlorine atoms alternately boned in a cube-shaped crystal lattice. Red dots are sodium atoms and green are chlorine atoms.  Example 2: Crystal lattice of iron The crystal lattice of iron consists of iron atoms arranged so that each atom represents one of the vertex of cube, plus an additional ninth iron atom is located in the very center of the cube. The blue balls are iron atoms  Example 3: Crystal lattice of zinc The crystal lattice of zinc consists of zinc atoms arranged so that each zinc atom represents one vertex of hexagonal prism, plus additional 13th and 14th zinc atoms are located in the very centers of the hexagons that form the upper and bottom bases of the hexagonal prism. The blue balls are zinc atoms.



Example 4: Crystal lattice of ice

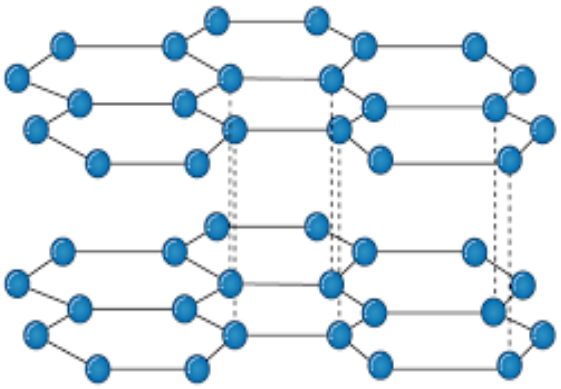
The crystal lattice of ice consists of water molecules. Each water molecule consists of one oxygen atom and two hydrogen atoms. In the crystal lattice of ice, water molecules are arranged so that six oxygen atoms (red) form a hexagon, with one oxygen molecule located at each vertex of the hexagon. The hexagons are connected to each other in one plane. In the structure of ice there are many such planes which are arranged parallel to each other in very close proximity. The hydrogen atoms are located between the two oxygen atoms and are not visible in the picture below. Water atoms when water is liquid or in a gaseous state (water vapour), are not arranged in the form of regular geometric bodies, as is the case when water is in the form of ice.

The red balls are oxygen atoms of water molecules.

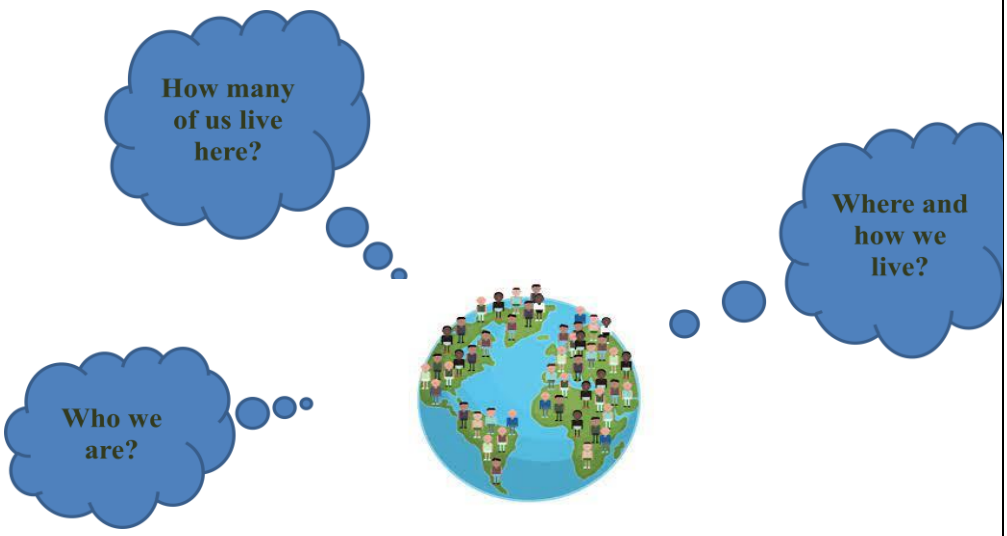
Example 5: Crystal lattice of graphite

The crystal lattice of graphite consists of carbon atoms. In the crystal lattice of graphite from which graphite pencils are made, carbon atoms are arranged so that six carbon atoms form a hexagon, with one carbon atom located at each vertex of the hexagon. The hexagons are connected to each other in one plane. In the structure of graphite there are many such planes which are arranged parallel to each other in very close proximity. The hydrogen atoms are located between the two oxygen atoms and are not visible in the picture below.

The blue balls are carbon atoms in a crystal lattice of graphite from which the pencil is made.

	 .
Opportunities to expand activities	Activity 5: During art class, pupils can choose which crystal lattice they want to draw (wooden crayons/crayons/watercolors) or make it with plasticine and toothpicks. When drawing/working with plasticine, take care that in some crystal lattice the atoms are the same (therefore they are the same color), and in others they are different (they should be different colors on the drawing/plasticine). Activity 6: During informatics class, pupils can choose which crystal lattice they want to draw using an available drawing program (e.g. Microsoft Paint).
Additional notes	/
Author	Marija Lesjak, Faculty of Sciences, University of Novi Sad, Serbia

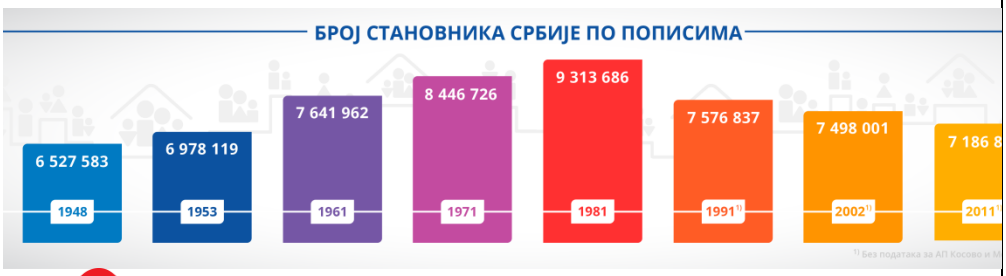
Title	**How is Belgrade growing?**
Key words	population/people, large numbers/millions/thousands, cities, countries
Short description	The scenario is based on making a link between very large numbers and demographic shifts. Students are informed that population shifts result from causes like births, deaths and migration. Students are given an example of a country whose population has fluctuated over time, for example Serbia, and asked to determine the difference in population between two censuses, provide commentary on the findings and consider the causes of these changes. Then, fluctuations in the population of major cities like Belgrade are used to illustrate large numbers (operations with thousands and millions). Students are tasked with determining how the population has changed over time using data provided for a given year (born during the year, died, immigrated and emigrated). The students are then given feedback on the world's largest cities and asked to rank them using the population number. Students also learn about the regions and countries of the world that are home to these urban centers. They can also practice comparing large numbers while making connections to the impact on the environment and way of life through comparisons of cities smaller than Belgrade. The significance of conducting a population census is highlighted as the teacher explains the concept to the class.
ICT tools included	*Programme Paint* *Interactive map of Serbia with the largest cities of Serbia by population* (https://serbiamap.net/index.html?mapId=12) *Interactive map of Serbia with Census 2011 population data* (population number, age-sex structure, families, householdls etc - for each selected location on the map; https://popis2022.stat.gov.rs/sr-Latn#)
Fields/ subjects	A1: Mathematics A2: Nature and society *A3: Art (indirectly)* *A8: Informatics (indirectly)*
Topics (for each selected subject)	A1: Addition and subtraction of numbers greater than a million; addition and subtraction of thousands and millions A2: Belgrade – the capital of Serbia; Larger cities in Serbia; Population of Serbia *A3: indirectly included - color matching (scale of color shades), color and size composition* *A4: indirectly included - introduction to the basic techniques of interactive online tools (search, zoom, information); drawing and coloring polygons in Paint in accordance with the comparison of the obtained data*
Expected prior knowledge of students	☞ *Recognizing, reading and writing large numbers - thousands and millions* ☞ *Bbasic arithmetic operations with large numbers - addition and subtraction* ☞ *Perception of Belgrade as the largest and capital city of Serbia* ☞ *Elementary knowledge of colors, shades, drawing and coloring* ☞ *Basic computer skills*
Expected outcomes	☞ Testing and improving the knowledge about the capital of Serbia, its population and the general population in Serbia ☞ Acquaintance with other large cities in Europe and the world and their number of inhabitants, changes in the population number; comparison of Belgrade with other big cities and Serbia with other countries, regions and the changes that are happening continuosly ☞ Determining the addition and subtraction of large numbers - thousands and millions, their comparison
Expected duration of the activity	2 school hours (90 minutes), with the possibility of extension to additional Art and i Informatics classes

Preparation of activities	Download the Power Point Presentation of the scenario and all the exercise documents via link https://inamath.uniri.hr/how-is-belgrade-growing/
Detailed description of all teaching activities	**Activity 1: Identifying and understanding the writing, reading, addition and subtraction of large numbers (thousands, millions and numbers greater than a million)** The teacher repeats with the students what they have learned about large numbers, their addition and subtraction, and then she/he discusses with the students what can be expressed in millions, giving some examples (money, distance between space objects). **Activity 2: Getting students familiar with Serbia's population and showing them how the population changes in Belgrade and anywhere else (with calculation examples)** The teacher uses the population as an example of how to deal with large numbers and tells them the story of the census. Students will learn more about how population data is collected, what the census is, and why it's important (in preparation for the 2022 Census). The census tells us how many people there are, where they live, how they live, how old or young they are, if they have ever moved, how many math professors, doctors, farmers, police officers, and so on there are. The population census will be done this year in the month of October. During that month, enumerators will go to every house in Serbia to get information. They will come to your house one day and ask your parents how old you are, what your parents do for a living, if you live with your grandparents, if you moved from somewhere else, etc. Why is it important for us? The census is a record for the future. It helps us plan the schools, colleges, and kindergartens we need, make plans for the progress of the country, write scientific papers about the population, find out where our grandparents came from, respect all the people who live in our area, whether they are Serbs, Hungarians, Slovaks, Croats, or anyone else, learn about their traditions, and much more.  After the Second World War, people in every country started to live better and longer, more and more children were born, countries got better, and the number of people in the world grew. The same thing happened in Serbia. Reading the numbers from the picture and coming up with questions about them.

How many people lived in Serbia in the year with the most people?

In 2011, when the last count was done, how many people lived in Serbia? (The last population count was in 2011, and there were 7,186,862 people living in Serbia at that time.)

Between those two census years, did the population change, if so, how much?



Србија
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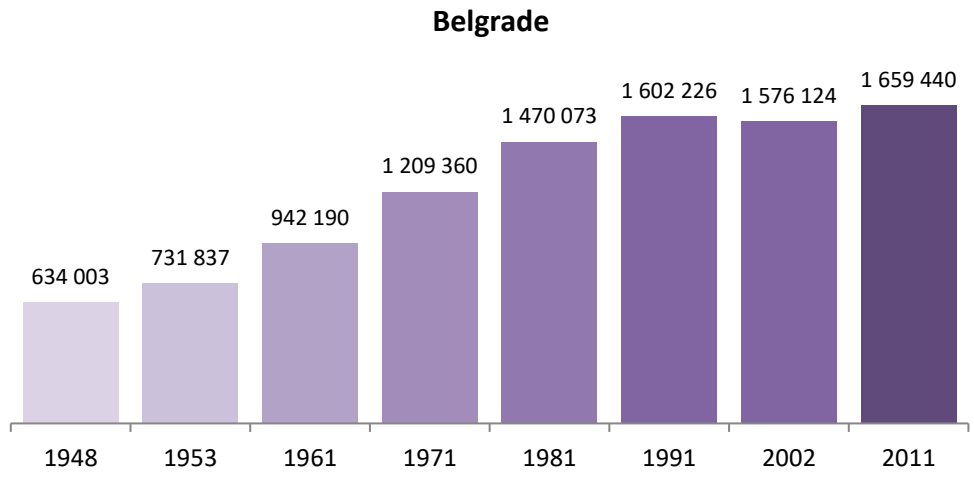
Activity 2.1: How is Belgrade growing?

Students repeat what they learned about Belgrade. Belgrade is the only city in Serbia that has the population over a million and that number is increasing every year (see picture). By how much did the population increase in 2011 compared to 1948?



So while Belgrade is experiencing population growth, Serbia is experiencing constant population decline? Understand why? Are you aware of the factors that cause a country's or a settlement's population to grow or decline?

With the students' help, the teacher explained that the population changes naturally, through births and deaths, and also through immigration and emigration, which are both called migrations. At the beginning of the year, the number of people living there is written down. During the year, the number of births, deaths, immigrants, and emigrants is also written down. The number of people living there at the end of the year is then calculated.

Before we get the results of this year's census, let's try to figure out how many people lived in Belgrade in 2021.

Population at the beginning of 2020	Number of births in 2020	Number of deaths in 2020	Number of immigrants in 2020	Number of emigrants in 2020	What is populati of Belgr at beginnin of 2021
1 694 480	17 236	25 526	43 428	38 562	**1 691 05**

*show to students the table with no final cell filled in

Talking about the final result. Since the 2011 Census, has the number of residents grown? If so, by how much? But compared to the year 2020, the population was still a little bit lower in 2021. This was because the world was in a special situation for the last two years. Why did so many people die in 2020? Aside from the fact that we have a lot of old people, the corona virus also had an effect on death rates.

Are more people moving to or away from Belgrade? Why is that, and who do you think is moving in? Are younger or older people? Why do people go to big cities like Belgrade and others?

Find out how many more people live in Belgrade than in Niš and Novi Sad, the next two largest cities.

Belgrade – population of 1 659 440
Novi Sad – population of 341 625
Niš – population of 260 237

Look with the students at the portals listed as ICT tools and show them how to select a specific city and read the information.

Activity 3: Introductiona nd learning about other big cities and countries in Europe and around the world

Do you know of any other large cities in Europe or the rest of the world that are over a million people? Which city in the world has the largest population, do you know?

Ten largest cities in the world by population:

Mumbai 20 185 064	**Beijing** 20 035 455	**Sao Paulo** 21 846 507	**Shanghai** 26 317 104	**Delhi** 29 399 141
Tokyo 37 435 191	**Osaka** 19 222 665	**Cairo** 20 484 965	**Dhaka** 20 283 552	**Mexico City** 21 671 908

Based on the data you received, write the names of the cities in the table, but by ordering them by population from largest to smallest. In the field next to each city, draw a circle of the appropriate size (the smallest circle next to the name of the city with the least population).

The possibility of connection with Rrt: paint the circles according to the same principle with shades of red.

Order	Name of the city	Population	Draw a circle	Country	Continent
1.					
2.					
3.					
4.					
5.					
6.					
7.					
8.					
9.					
10.					

*download the separate document with the table

Homework (may also be done during the class, depends on the time): Find these cities on the world map and write in the table in the remaining empty fields the country they are located in, and then the continent they are located on. Which continent has the most cities that are also the most populous in the world? Possibility of connection with ICT tools, use of Google Maps.

Activity 4: Questions and discussion about why people move to big cities and how this affects urban and rural life and the environment?

The teacher starts a discussion with the students about rural-urban migration, Serbian villages becoming empty, the elderly population remaining, young people leaving, and changes. Students' active involvement in these discussions.
Why do people relocate from villages to cities so frequently?
What happens to the villages and how does it impact the cities?
How does the environment change as more people live there (are they drawn to factories, traffic, crowds, etc.)?

Possibilities for expansion of activities

Possibility of connecting activities with Art:

Color matching and color composition. Students can color certain countries or cities of Serbia with shades of the same color, from lighter to darker, after comparing the population. They can also be given painting topics for independent work on migration, the impact of large numbers of people on the environment, the differences between city and country life, and the like.

Possibility of connecting activities withInformatics:

Drawing and coloring in Paint or a similar program on a background that can be a map of Serbia with its major cities is a good way to learn how to use a computer. Students can use an online interactive map as a way to practice zooming, moving the map, searching, and reading about certain places.
The topic can also be used to compare smaller numbers (like villages and smaller towns), so that by the end of all the lessons, students will have the same link between nature, society, and math. This will help them remember cities because they will have the same associations with them.

Additional notes

If a student attends classes according to the IOP1 or IOP2 program, it is necessary to include a personal companion/assistant in the preparation, and choose activities in which it is possible to include the student as much as possible.

Authors

Milica Solarević, PhD, Associate Professor, UNS

Title	**Historymathics**
Keywords	*Natural numbers greater than 1000,* *The notion of Century,* *Historical figures*
Short description	The main topic of this scenario is connecting lessons from Nature and Society dedicated to history (famous historical figures and events) with the addition and subtraction of four-digit numbers. The core part of this scenario is directed at scientists and explorers, while the extended version of this scenario can include national historical figures and events.
ICT tools included	Drawing software (Paint) Scratch
Areas (select)	A1: Mathematics A2: Science - History A3*: History – National History A4*: Computer science A5*: Art *possibility of expanding activities
Topics (for each selected area)	A1: Practiceing operations with numbers greater than 1000
	A2: Historical figures and events from science
	A3*: Historical figures and events from the national history
	A4*: Introduction to the drawing software and Scratch
	A5*: Getting to know certain painting techniques
Expected prior knowledge of students	Knowledge of basic arithmetic operations with natural numbers up to 1000. *Elementary knowledge of computer work
Expected outcomes	Determination of arithmetic operations with natural numbers greater than 1000. Adoption of the notion century. Getting to know historical figures and events.
Expected duration of the activity	2 school hours, with the possibility of extending it to a thematic day dedicated to national history
Preparation of activities	Download a pdf/ppt of the introductory text, download pdf of graphic representations for activities 4 and 6: https://inamath.uniri.hr/historymathics/ Internet access for activity 3. * Internet access for activities 7 and 9. *https://scratch.mit.edu/help/videos/
A detailed description of all teaching activities	**Activity 1: Introductory text projected on a "smart" board.** *Petra (the classroom Mr. Grumble, going through a book from History and grumbling to himself):* Alas, years, names, years, names, and more years and years... this is going to be a very tiring lesson. *Sofia (Petra's best friend, and as it usually happens, the most cheerful person in the class, they have been sharing a desk since the second grade):* Don't grumble, it's not that bad. At least it makes some sense, we're talking about people and events. Not like your favorite math where they pester us with numbers for no reason. Add them up, subtract them, then find x, as it always gets lost...ugh. Well, that's hard. *Petra and Sofia are so engrossed in criticizing the curriculum that they don't notice teacher Peter standing above them and smiling.*

Teacher Peter: Actually, you are both right. Let's try to combine the most fun parts of both subjects and make a new one.
Sofia (brightly): Yes! And let it be called **Historymathics**!
Petra (with a grunt): Yes, yes, the fun parts... as long as it doesn't turn out the other way around...

Activity 2: Historical figures from sciences - preparatory activity. Together, the teacher and the children make a list of historical figures to be studied. The choice of historical figures and events may differ from country to country, depending on what the curriculum provides. Scientists, researchers, and inventors are chosen for this activity for their international character.

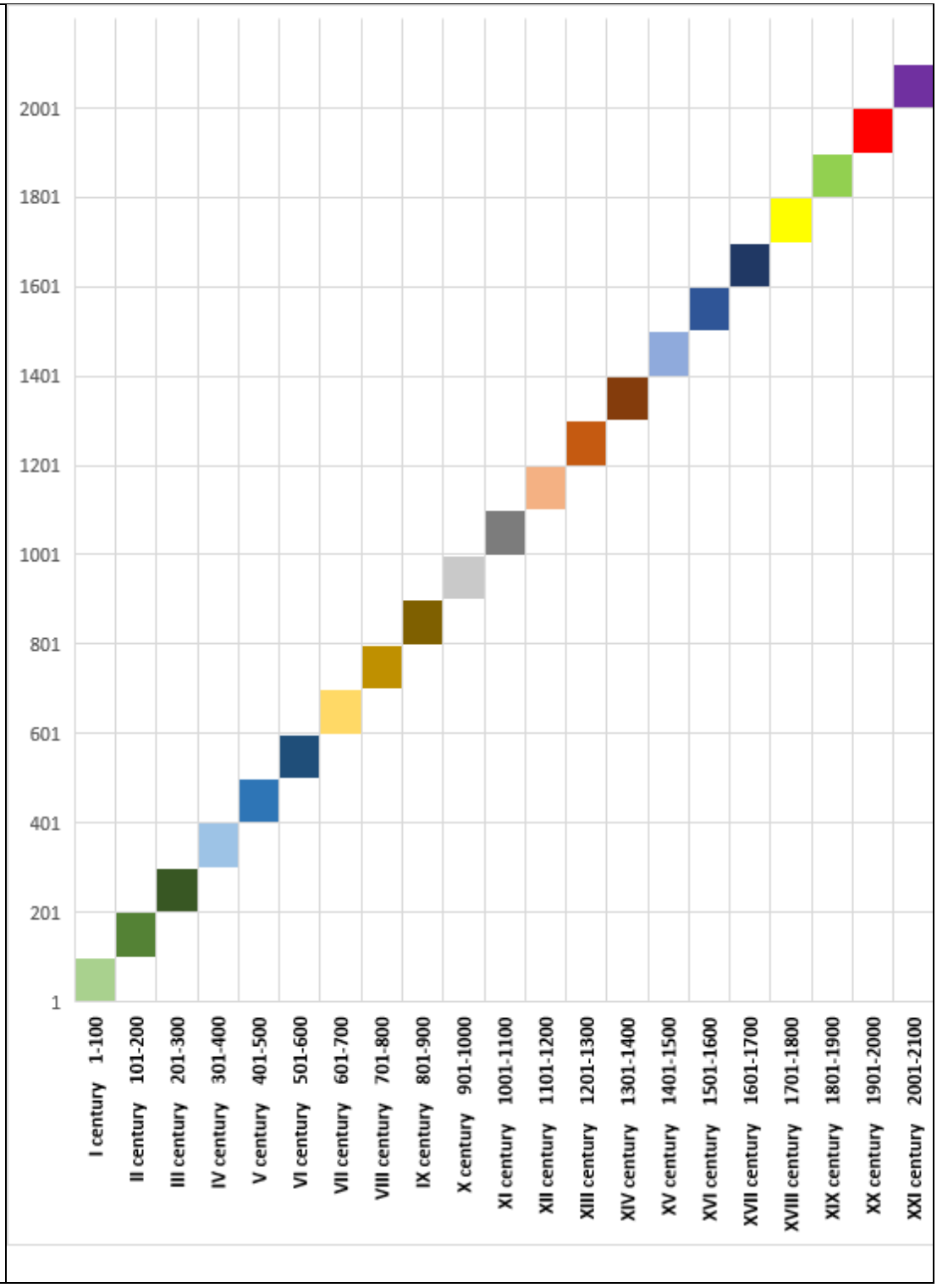
Petra: And does it have to be someone who lived a very, very long time ago? Because I have a name too! Maryam Mirzakhani! She is the first woman recipient of the Fields Medal. It is …
Teacher Peter: I know, something like the Nobel Prize for mathematics, right? We can put her on the list too.
Sofia: When it comes to choosing, I want Amelia Earhart! She was fearless!

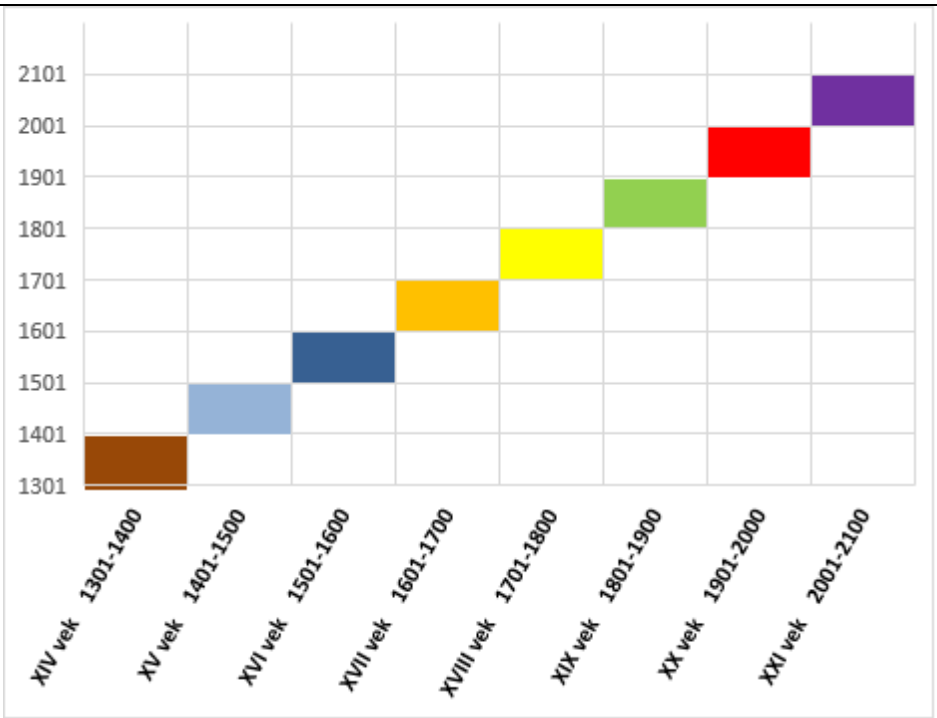
Teacher Peter (with a smile): Agreed!

Activity 3: Research. Under the supervision of the teacher, the children research the personalities from the list, year of birth, year of death, and achievements by searching the Internet. On the "memory" board, they fill in the attached table together.

	Year of birth	Year of death	Century (vs. year of birth)	Achievement
Christopher Columbus				
Leonardo da Vinci				
Galileo Galilei				
Isaac Newton				
Andres Celsius				
Charles Darwin				
Thomas Edison				
Nikola Tesla				
Marie Curie				
Albert Einstein				
Alexander Fleming				
Amelia Earhart				
Maryam Mirzakhani				

Teacher Peter: A really colorful company. Let's place them in the appropriate time period. Let's see who lived in which century. Do you remember what a century is? A period of 100 years. The first century begins in the 1st year and ends exactly in the 100th year, the 101st year is the beginning of the 2nd century, and the last year of the 2nd century is the 200th, and so on.


2001
1801
1601
1401
1201
1001
801
601
401
201
1
I century 1-100
II century 101-200
III century 201-300
IV century 301-400
V century 401-500
VI century 501-600
VII century 601-700
VIII century 701-800
IX century 801-900
X century 901-1000
XI century 1001-1100
XII century 1101-1200
XIII century 1201-1300
XIV century 1301-1400
XV century 1401-1500
XVI century 1501-1600
XVII century 1601-1700
XVIII century 1701-1800
XIX century 1801-1900
XX century 1901-2000
XXI century 2001-2100



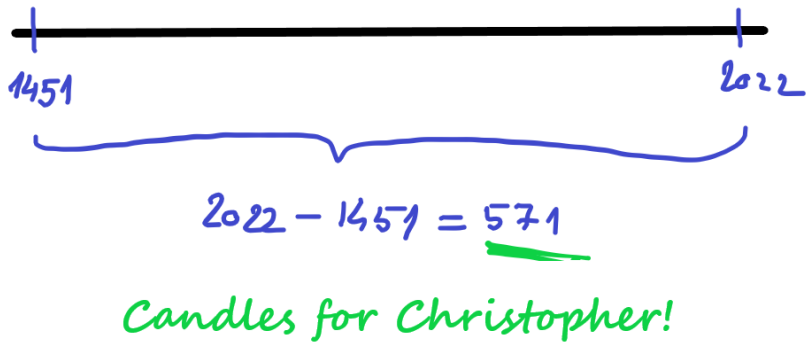
Activity 4: Timeline. On the "smart" board, a part of the timeline that includes the birth years of the mentioned persons is projected. Centuries are marked with different colors. Children place the selected personalities in the appropriate age according to the year of birth. We are discussing the year of death, i.e., did it go into the next century.

	Year of birth	Year of death	Century (vs. year of birth)	Achievements
Christopher Columbus	**1451**	**1506**	**XV**	discovery of America
Leonardo da Vinci	**1452**	**1519**	**XV**	Last Supper, Mona Lisa, applied mechanics
Galileo Galilei	**1564**	**1642**	**XVI**	telescope
Isaac Newton	**1643**	**1727**	**XVII**	law of gravitation
Andres Celsius	**1701**	**1744**	**XVIII**	Celsius thermometer scale
Charles Darwin	**1809**	**1882**	**XIX**	theory of evolution
Thomas Edison	**1847**	**1931**	**XIX**	sound recording, light bulb, direct current
Nikola Tesla	**1856**	**1943**	**XIX**	alternating current
Marie Curie	**1867**	**1934**	**XIX**	x-rays
Albert Einstein	**1879**	**1955**	**XIX**	theory of relativity
Alexander Fleming	**1881**	**1955**	**XIX**	penicillin
Amelia Earhart	**1897**	**1937**	**XIX**	the first woman to fly solo across the Atlantic Ocean
Maryam Mirzakhani	**1977**	**2017**	**XX**	mathematics

Activity 5: Birthday math - anything is possible. We practice calculation operations with four-digit numbers.

Teacher Peter: Now imagine that they all found the Philosopher's Stone and learned how to make the Elixir of Life (We all read the first Harry Potter book, right?).
1. How many candles on the cake would each of them have this year? How many candles is that in total?
2. How old would Leonardo have been the year Amelia was born?
3. Let's assume that the year is 1921 and that Alexander, Albert, Nikola, Thomas, Marie, and Amelia met. Calculate the following: we multiply the difference in the ages of Albert and Alexander by the ages of Marie, and then add the product of the years of Nikola and Thomas, and Amelia.
4. How much older is Galileo than Isaac?
5. How old would Christopher be in the first year of the 17th century? And how much in the last years of the 20th century?
6. If Charles had met Maryam in the first year of the 21st century, how many years would they have together?
7. What year would Andres celebrate his 342nd birthday? In what century?

Activity 6: Graphic representation. With the help of a drawing program, children illustrate previous problems with appropriate timelines. They are divided into groups, each group gets one of the problems to illustrate, and then presents it to the others. They are getting familiar with working on a graphic board.



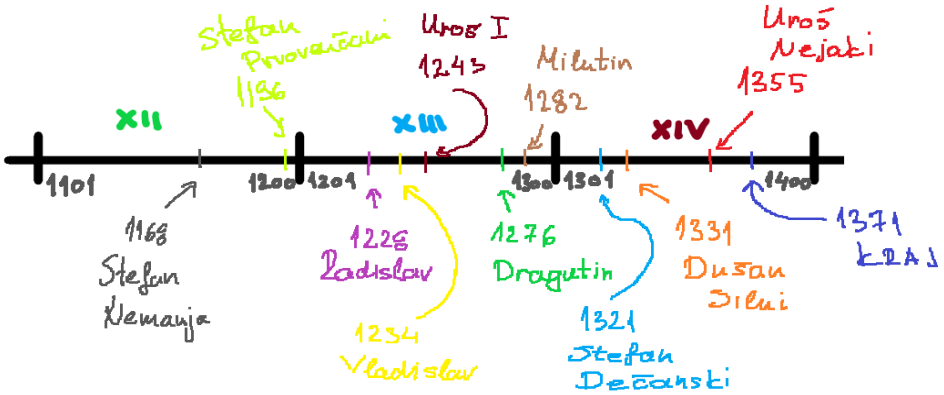
Expanding activities

The possibility of organizing a thematic historical day:

Activity 7: Group work. With the help of textbooks, students create a list of historical figures that are covered in one teaching unit. Together, they draw a timeline for the observed period on the board. An example is given for the Nemanjić Dynasty.

Name	Interesting fact	year of the reign
Stefan Nemanja	the great prefect - the founder of the dynasty	1168-1196
Stefan Nemanjić Prvovenčani	the first king	1196-1228
Stefan Radoslav Nemanjić	copper coins	1228-1234
Stefan Vladislav Nemanjić		1234-1243
Stefan Uroš I Nemanjić	silver coins	1243-1276
Stefan Dragutin Nemanjić		1276-1282
Stefan Milutin Uroš II Nemanjić	expanded the country	1282-1321

	Stefan Uroš III Dečanski		1321-1331
	Stefan Dušan Uroš IV Nemanjić	Dušan the Mighty - emperor	1331-1355
	Stefan Uroš V Nemanjić	Uroš the Weak - the last ruler from the Nemanjić dynasty	1355-1371



Activity 8: Quiz. Students are divided into four teams. Each team compiles five questions based on Activity 5. Each question is written on a separate piece of paper. All questions are placed in one container, mixed, and each team draws three brain teasers. The first team to solve all three puzzles correctly wins and they have the right to choose the topic of the next activity.

Activity 9: Under the supervision of the teacher, search the Internet to find interesting facts about flags, weapons, jewelry, and ornaments from the period of history from Activity 7, and the winning team from Activity 8 chooses a topic for the art class.

In agreement with informatics teachers, older students mentor younger students:
Activity 10: Making simple scratch animations that illustrate puzzles from activities 5 and 8.

Additional notes	If a student attends classes according to an individual educational program, it is necessary to include a personal companion in the preparation and choose activities in which it is possible to include the student as much as possible.
Authors	Ivana Štajner-Papuga

Title	**Mathematics of small and big ones**
Key words	Natural numbers up to and greater than a million Multiplication and division Relationships between units of mass and length Comparing numbers up to and greater than a million Plants Animals Solar system
Summary	This scenario aims to connect knowledge about animals/plants/astronomical objects with lessons from mathematics reagarding comparing numbers up to and over a million - smaller/bigger, shorter/taller, lighter/havier, and how many times. Pupils will get tasks to compare how much lighter/havier and shorter/taller, is an animal based on data on animal mass in different units (g, kg, t) and animal height (mm, cm, m), respectively,. Also, the same type of tasks will be applied to the height and mass of plants and their fruits, as well as to the size of astronomical objects from the solar system. Additionally, pupils will be asked to draw/make selected animals/plants/astronomical objects of different sizes in a scale as realistic as possible.
ICT tools included in scenario	*Drawing program (Microsoft Paint)
Areas	A1: Mathematics A2: Science A3*: Arts A4*: Informatics
Topics (for each selected Area)	A1: Writing, reading and comparing numbers up to and greater than a million; Relationships between smaller and larger units of mass and length;
	A2: Plants, Animals, Solar system
	A3*: Practicing different techniques of drawing with wooden crayons/crayons/watercolors/markers or working with plasticine.
	A4*: Drawing on computer using an available drawing program (e.g. Microsoft Paint)
Expected prior knowledge of pupils	Writing, reading and comparing numbers up to and greater than a million; Mass and length and their units, as well as the relationship between units; Elementary knowledge of animals, plants and astronomical objects (Solar system); *Basic knowledge regarding drawing techniques and working with plasticine; *Basic knowledge regarding working on computer.
Expected outcomes	Established knowledge on mathematical operations with natural numbers up to and greater than a million (multiplication and division); Establishing knowledge on relationship between different units of mass and length; Extended knowledge on animals, plants and astronomical objects (Solar system).
Expected duration of the activity	2 school hours (90 mins in total), with the possibility of extension to additional art and computer science classes
Preparation activities	/

Detailed description of activities	Activity 1: Units of mass and length and their ratios. The teacher and the pupils repeat measurement units for mass (g, kg and t) and length (mm, cm, m, km) and repeat the mutual relations between them (1 t = 1 000 kg = 1 000 000 g; 1 km = 1 000 m = 100 000 cm = 1 000 000 mm). Activity 2: ZOO Mathematics The teacher presents the masses and heights of different animals to the pupils and gives pupils the task to calculate how many times the presented animals are smaller/bigger and shorter/taller between each other. Examples of animal masses: Blue whale - 150 t Elephant - 5 t Dolphin - 208 kg Man - 70 kg Dalmatian dog - 35 kg Fox - 4 kg Sparrow - 30 years old Bat - 2 years old Examples of height/length of animals: Blue whale - 27 m Anaconda - 9 m Giraffe - 6 m Crocodile - 5 m One-humped camel - 2 m Man - 170 cm Squirrel - 25 cm Fly - 4 mm Activity 3: BIO mathematics The teacher presents the height of different plants and the mass of plant fruits to the pupils and gives them task to calculate how many times the presented objects are smaller/bigger, taller/lower between each other. Examples of plant fruit mass: Watermelon - 12 kg Pumpkin - 7 kg Black pine cone - 300 g Pomegranate - 250 g Plum - 50 g Cherry - 11 g Raspberry - 5 g Hazelnuts 1 g Examples of plant height: Sequoia - 90 m Pine - 20 m Walnut - 15 m Pear - 5 m Lilac - 4 m Potatoes - 100 cm Dandelion - 25 cm Violet flower - 20 cm

	Activity 4: ASTRO Mathematics Teacher presents to the pupils the diameters of the astronomical objects from the Solar system and gives them task to calculate how many times the presented objects are smaller/larger than each other. Sun – 1,391,000 km Jupiter – 140,000 km Saturn – 117,000 km Neptune – 50,000 km Uranus – 51,000 km Earth - 13,000 km Venus – 12,000 km Mars – 7,000 km Mercury – 5,000 km
Opportunities to expand activities	Activity 5: In art class, pupils can choose which animals/fruits/astronomical objects they want to draw (wooden crayons/crayons/watercolors/markers) or make with plasticine. The challenge is to draw/make selected objects in as realistic a scale as possible, based on the calculation of how many times something is bigger/smaller, higher/lower. Activity 6: During informatics class, pupils can choose which animals/fruits/astronomical objects they want to draw using an available drawing program (e.g. Microsoft Paint). The challenge is to draw selected objects in as realistic a scale as possible, based on the calculation of how many times something is bigger/smaller, higher/lower.
Additional notes	/
Author	Marija Lesjak, Faculty of Sciences, University of Novi Sad, Serbia

Title	**Musical fractions**
Keywords	Fractions Note duration
Short description	Lesson links Note durations and Fractions (reading, writing, and comparing) Through the game of stepping and musical exercises, children repeat (intuitively adopt) the concepts of whole, half, quarter, and eighth notes and their mutual relationships. With the help of music, note durations are associated with fractions.
ICT tools included	Drawing software (Paint) Scratch
Areas (select)	A1: Mathematics A2: Music A3*: Art A4*: Informatics *Possibility of expanding activities
Topics (for each selected area)	A1: Getting to know the concept of a fraction
	A2: Determining the duration of notes
	A3*: Getting to know certain painting techniques
	A4*: Introduction to the drawing program and Scratch
Expected prior knowledge of students	Knowledge of arithmetic operations with natural numbers up to 100 *Elementary knowledge of computer work
Expected outcomes	Understanding the concept of a fraction, understanding the procedure of comparing fractions with the same denominator or the same numerator.
Expected duration of the activity	2 school hours, with the possibility of extending it to an additional art lesson, as well as a themed day - a day of happy fractions
Preparation of activities	Download the pdf/ppt file of the introductory text. Download the accompanying mp3 files for the introductory text. Download pdf file with illustrations. https://inamath.uniri.hr/musical-fractions/ *https://scratch.mit.edu/help/videos/
A detailed description of all teaching activities	**Activity 1: An introductory text projected on a "smart" board.** *Teacher Ljubica lost in her thoughts enters the classroom and hums to herself Orf's composition O Fortuna (mp3 file).* Peter*, the class clown, and a big fan of rock music looks at the teacher with interest and slowly starts tapping on the table following the rhythm (mp3 file). The teacher smiles at Peter as she places the books on the desk and greets the class.* *Teacher Ljubica:* Good afternoon, children! Peter, congratulations, you caught the rhythm perfectly. *Peter (satisfied):* I know what you were humming! Carmin! *Teacher (with a growing smile):* You are very close. Carmina Burana. Actually, a song called O Fortuna. Composed by Karl Orff, but you'll learn about that when you're a little bit older. *Peter:* It's not easy to follow that rhythm, it goes at a steady speed for a while, then it gets slower...

Teacher: Right. You made a good point. Kids, do you remember that last year we learned about note durations?

A weak, unconvincing yeeees comes from the classroom.

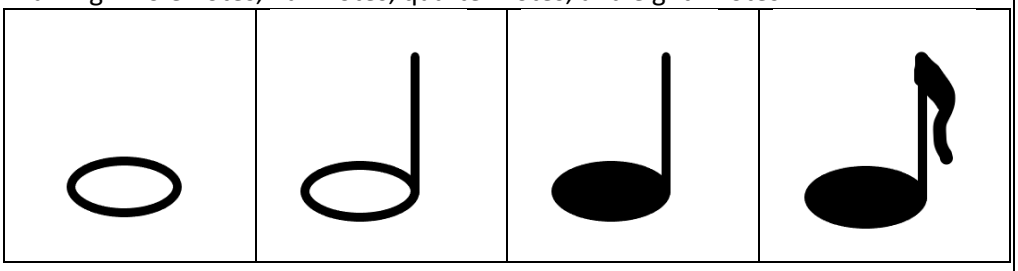
Teacher: Good, so it's time to repeat it. It fits perfectly with today's topic in mathematics.

Peter (in a panic): Mathematics!?! How?! Why math?! I love music, but math....it will ruin everything.

The teacher (now laughing a lot): Oh, Peter, Peter, music is mathematics.

Activity 2: Note durations.
Repeating the duration of the notes with the help of beats and steps, while pronouncing the syllable "la". First, everyone pronounces the syllable "la" together with four even strokes of the pencil on the table, and thus we illustrate the duration of the whole note. Then we illustrate a half note with two beats and a quarter note with one. Children are divided into groups of five. Four of them form a circle, and the fifth member of the group goes around them singing the syllable "la". A full circle corresponds to a whole note, passing just two friends a half note, and passing one a quarter note.

Activity 3: Labels.
Drawing whole notes, half notes, quarter notes, and eighth notes:



Children are divided into two groups, and they assign tasks to each other. Both groups come up with sequences of 15 notes of different duration, exchange tasks, and prepare an interpretation of the given sequence. One person in the group is the conductor in charge of the rhythm (tapping on the table), and the others sing like a real choir.

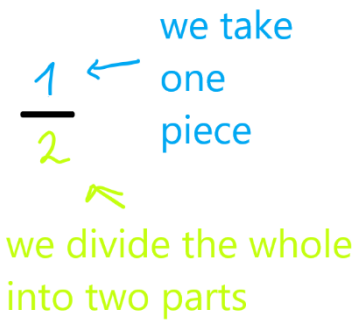
Teacher (now slightly overwhelmed by all that noise): This was a wonderful noise! Perfect! Just so you know, you played pure math!

Peter (completely sure of himself): Of course not.
Teacher: Let me convince you that you did. Let's start with the whole note. You noticed her name - whole. Do you agree that, because it is a whole, I should assign the whole number 1 to it, exactly 1?

Peter (suspiciously): Good...

Teacher: Now tell me one important thing. While Peter plays one whole note, how many of these halves can Nina play?

Nina (Peter's best friend and, unlike him, a big fan of math)*:* I'm always in favor of experimental verification, but I think I know the answer right away. Those two notes with the neck. That Peter's whole note lasts four beats, and this one of mine

	lasts only two, so I can manage to play two while Peter stretches out that one. Am I right? *Teacher:* You are. That's right. That's why your note is called half. Do you know how to write that? $\frac{1}{2}$ It's a fraction. That is our topic today. The dash is called the fractional line, and the number two below it tells us that we have divided our whole into two equal parts. The one above the line tells us that we took only one of those two equal parts.  We call the number above the line **the numerator** because it counts how many pieces we take, and the number below the line is **the denominator** because it tells us in how many pieces, we must divide our whole. *Peter:* Understood! This means that for a quarter below that line, the number should be four! The whole note is four beats, and for a quarter we need only one, so we divide the whole note into four parts and take only one of them. *Teacher:* Well done! Just pay attention that we are dividing the whole note into four equal parts. *Nina:* And if we play three of those black notes with the neck, does that mean we took three pieces of the whole note? *Teacher:* That's right. *Nina:* I think I know how to write it down $\frac{3}{4}$ **Activity 4:** Write the note durations given in the previous activity as fractions. *Nina:* And to me, the most beautiful note is the one with a small tail. *Peter (very officially):* It's not a small tail, it is a flag on a flagpole. *Nina (slightly offended, grumbles):* If it looks like a tail, it is a tail... *Teacher:* Please don't argue. It is called a flag. And it is not a flagpole, but a stem... That pretty picture marks the one-eighth duration. Can someone explain to me what that means?

Nina and Peter in unison: We split the whole note into eight parts and take one!

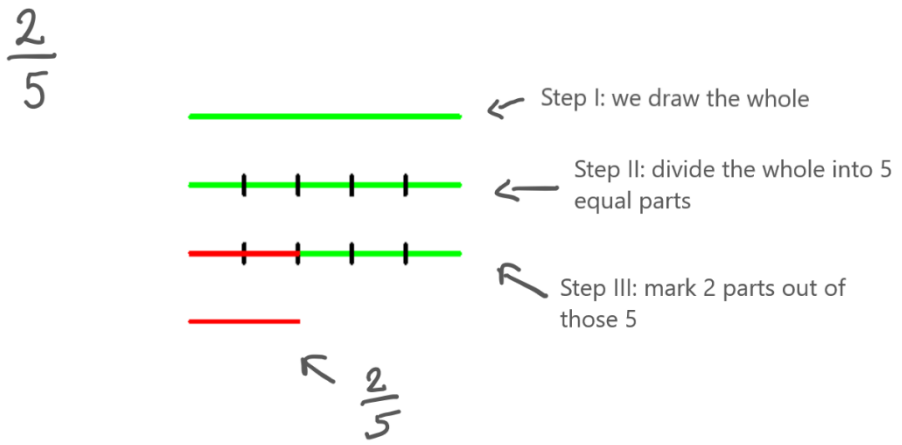
Teacher (satisfied): You understood everything. And we write with

$$\frac{1}{8}$$

However, fractions do not appear only in music. They are all around us. Here is an example. Imagine that we have a plum cake that is divided into nine pieces and Petra takes one. How much did Peter take?

Nina (like out of a cannon): $\frac{1}{9}$! But it's Peter. He doesn't even start without two pieces. *Peter (satisfied):* That's right, $\frac{2}{9}$ or nothing. 😊

Activity 5: We draw fractions. The teacher assigns ten fractions to be represented graphically as in the attached drawing. Depending on the equipment of the classroom, it is also possible to use a computer and available drawing software.



Activity 6: We listen to fractions. Children listen to a selected composition and record the duration of the notes as they hear them using dashes of different lengths. Proposal of compositions

- Strauss – “Also Sprach Zarathustra”

https://www.youtube.com/watch?v=dfe8tCcHnKY

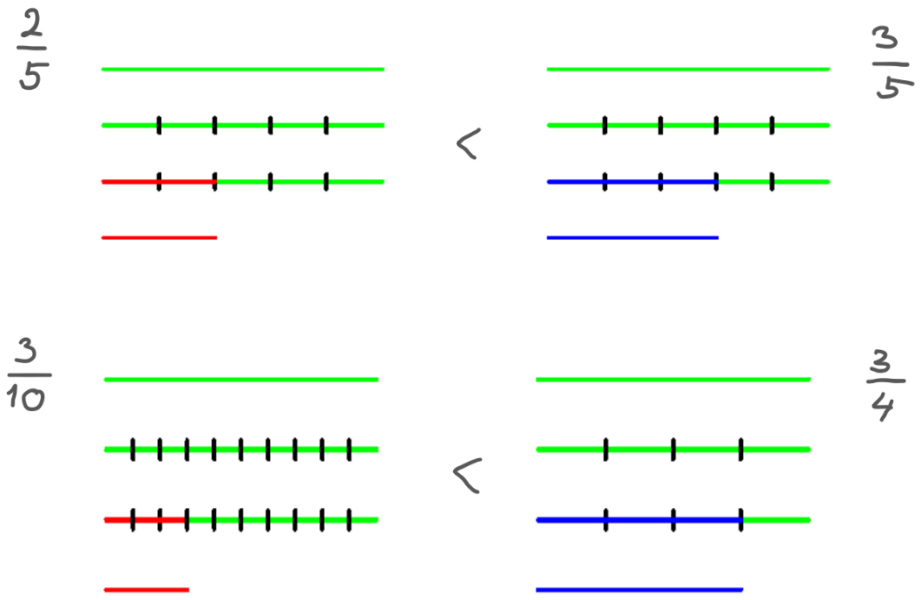
- Greig - "In the Hall of the Mountain King"

https://www.youtube.com/watch?v=4nMUr8Rt2AI

Group work is allowed and discussion about dash lengths is encouraged, i.e. what would correspond to a whole note, and what half a note,... All solutions are correct. 😊

Activity 7: Let's compare fractions with music.
Let's divide the children into two groups.
• The first group plays a quarter note, and the second group plays a quarter note three times. We discuss what took longer.
• The first group plays a quarter note three times, and the second group plays a half note three times. We discuss what took longer.

Nina: Of course! One quarter is less than three quarters! For that one quarter, we took only one piece after splitting, and for three quarters as many as three such pieces!

	Peter (excitedly, interrupting Nina): And the second one makes sense too! Well, a half note is longer than a quarter note, we play it longer! And if we take three longer pieces, it must be bigger than three shorter pieces! And I noticed something else! The higher the number below the line, the smaller the pieces! **Activity 8: Let's compare fractions by drawing.** Using the fraction drawing technique illustrated in activity 4, children compare fractions that have the same denominator or the same numerator.  *Peter (very cheerfully):* Well, this is great! Next time my parents scold me for listening to music too loud I can tell them I'm practicing fractions! 😊
Expanding activities	The possibility of organizing a themed day: **Activity 9:** Children illustrate the music from activity 5 using different painting techniques. In agreement with informatics teachers, older students mentor younger students: **Activity 10:** Depending on the software available in the school, draw notes of different durations and introduce students to working on graphic boards. **Activity 11:** Creating simple scratch animations illustrating note durations, as well as examples from activities 4, 6, and 7. https://scratch.mit.edu/projects/698228423 **Activity 12:** Under the supervision of the teacher, search the Internet to find interesting facts about the compositions that were listened to during the lesson.
Additional notes	If a student attends classes according to an individual educational program, it is necessary to include a personal companion in the preparation and choose activities in which it is possible to include the student as much as possible.
Authors	Ivana Štajner-Papuga

Title	**Mathematical kitchen**
Key words	Natural numbers up to 1000 Calculation operations - Addition, multiplication and division Fractions Measurement of mass and volume Healthy diet
Summary	This scenario connects the process of food preparation with calculation operations and measurements of mass and volume. Pupils will get task to calculate how many individual ingredients (mass and volume expressed in standard and nonstandard units) they need in order to make certain healthy foods for the whole class. Also, based on the price of an individual food items, pupils will calculate how much money they need in order to buy and prepare the selected healthy food for the whole class. After that, by measuring the mass and volume of the ingredients they will make chosen healthy foods during the class and taste it. Finally, based on the data on how many calories certain amount of food item has, they will calculate how many calories they consumed during this math lesson. Afterwards, they will use that data to calculate how many exercises (squats, push-ups, etc.) they have to do in order to burn off the calories they have taken in, and they will do those exercises during class. During the class, the concept of healthy diet and lifestyle should be explained to the pupils, pointing out that it is very hard to balance daily calories intake and physical activity in order to avoid weight gain.
ICT tools included in scenario	*Online calculators of the number of calories per ingredient, such as https://www.nhs.uk/live-well/healthy-weight/managing-your-weight/calorie-checker/; https://www.mayoclinic.org/healthy-lifestyle/weight-loss/in-depth/calorie-calculator/itt-20402304 *Online calculator of the amount of food expressed in nonstandard units, such as ½ cup, tablespoon, teaspoon, etc., such as https://goodcalculators.com/cooking-conversion-calculator/
Areas	A1: Mathematics A2: Science A3*: Physical activity A4*: Informatics
Topics (for each selected Area)	A1: Natural numbers up to 1000; Calculation operations - Addition, multiplication and division; Fractions; Measurement of mass and volume.
	A2: Healthy diet
	A3*: Aerobic and anaerobic exercises
	A4*: Introduction to online calculators
Expected prior knowledge of pupils	Established knowledge on mathematical operations (addition, multiplication and division) with natural numbers up to 1000; Established knowledge of a fraction; Established knowledge of volume and mass and their units; *Basic knowledge regarding working on computer.
Expected outcomes	Deepened knowledge on mathematical operations with natural numbers up to 1000; Deepened knowledge on fraction and mathematic operations with fractions; Deepened knowledge on measuring mass and volumes and units; Deepened knowledge on healthy nutrition
Expected duration of the activity	2 school hours (90 mins in total), with the possibility of extension to classes of physical activity and thematic days related to health.
Preparation	Get the necessary ingredients depending on the chosen recipe;

activities	Get the kitchen equipment for making food depending on the chosen recipe. Additional material for implementation of this scenario please download from the link - https://inamath.uniri.hr/wp-content/uploads/2022/12/Mathematical-kitchen-ENG-presentation.pptx
Detailed description of activities	Activity 1: Review on standard and nonstandard units for mass and volume and familiarization with equipment for measuring The teacher repeats with the pupils the standard measurement units for mass (g and kg) and volume (mL and L) and introduces them to the equipment for measuring mass and volume in the kitchen (kitchen scales, containers for measuring volume, etc.) followed by demonstration done by pupils on how to use them. The teacher introduces the pupils to a nonstandard unit for mass and volume that are routinely used during cooking - cup, tablespoon, tea/coffee spoon, pinch, drop, dL, and points out to the students the relationship between measurement units from the SI system and nonstandard units. Activity 2: Recipe 1 and calculating of the required ingredients The teacher introduces the pupils with a recipe that uses standard units for mass and volume, such as mL, L, g and kg. Pupils get a task to calculate how many ingredients they need if they want to prepare a meal for the whole class. Example - Healthy Pancakes: 80 g of oat flakes 70 g of buckwheat flour 10 g of olive oil 3 g salt 4 g of baking powder 4 g of cinnamon 4 mL of vanilla extract 2 bananas 200 mL of milk 5 mL of sunflower oil This recipe is for 12 pancakes. Calculate how much of each ingredient is needed in order for each child in the class to eat 2 pancakes? Activity 3: Recipe 2/3 and calculating of the required ingredients The teacher introduces the pupils with a recipe that uses nonstandard measuring units, such as a tablespoon, a tea/coffee spoon, ½ cup, a pinch, a drop, etc... Pupils get a task to calculate how many ingredients they need if they want to prepare a meal for the whole class. Example 1 - Smoothie with Forest fruit: 2 dL of yogurt 2 dl of milk 3/2 cup of forest fruits berries 2 teaspoons of honey 1 half teaspoon of cinnamon This recipe is for smoothie for two people. Example 2 – Lemonade with honey 1/5 cup of squeezed lemon juice 200 mL of water 1 teaspoon of honey This recipe is for lemonade for one person.

	Calculate how much of each ingredient is needed in order for each child in the class to get one smoothie or lemonade? Calculate the amount of ingredients in nonstandard units. After that, based on the obtained values, express the amount of the ingredient in measures from the SI system. Transferring values from nonstandard units to SI units can be done using an online converter of the nonstandard units. Activity 4: In the presence of the teacher and with the help of the available kitchen equipment, recipe and calculations they made, pupils will make selected food and serve it to the whole class. Activity 5: While the pupils are enjoying in the food they have made, the teacher should introduce them to the concept of a healthy diet, which is based on a higher intake of fruits, vegetables, whole grains, meat that is not fatty (fish) and plenty of fluids in the form of water and freshly squeezed juices, and on low intake of sweets and salt.
Opportunities to expand activities	Activity 6: During the math class, based on the selected food that was prepared and the number of pupils who consumed it, pupils get task to calculate the total cost price of the selected food stuffs that is prepared based on the current prices in retail stores. Activity 7: During theme days at school, such as a school bazaar or a health/healthy eating day, pupils can prepare food for visitors based on the calculations made in class. It is preferable to prepare foods from the healthiest ingredients possible - such as pancakes with buckwheat flour and banana, lemonade with honey, smoothies made from seasonal fruits with nuts, porridge made from oats, quinoa, nuts and almond milk, etc. In addition, students can calculate the number of total calories per portion and compare the calories of the healthy foods they have prepared with the calories of less healthy foods that are commonly consumed, thus pointing out the importance of choosing healthy foods for health. Calories can be calculated using an online calculator. Activity 8: During classes of physical activity, teacher should introduce pupils to the concept of calorie as a measure of how much energy a food contains and why the amount of daily calorie intake is important for health. Based on the calculated total calories that the selected food portion has, calculate how long the student should run / how much distance they should run / how many times they should do some exercise in order to use the calories they took with a meal. After that, pupils should do calculated exercises. In this way, pupils will see direct connection between healthy nutrition and regular physical activity. Calories can be calculated using an online calculator.
Additional notes	/
Author	Marija Lesjak, Faculty of Sciences, University of Novi Sad, Serbia

Title	**Is there something hidden in circles?**
Keywords	Circle, Diameter, Radius, Circumference, Number Pi
Short description	With the help of a dramatic text, and through a large selection of examples from the real world and games, students are introduced to the number pi and its role in nature. Through play and artistic expression, we determine the terms circle, circle, and radius. On an intuitive level, we introduce the concept of the circumference of a circle and go beyond the set of natural numbers.
ICT tools included	Drawing software (Paint) Scratch Micro:bit
Areas (select)	A1: Mathematics A2: Art A3: Literature A4*: Computer science A5*: History *possibility of expanding activities
Topics (for each selected area)	A1: Circle and radius
	A2: Getting to know certain tempera painting techniques
	A3: Reading and understanding the dramatic text
	A4*: Elementary programming in Scratch and Micro:bit
	A5*: Archimedes and Einstein
Expected prior knowledge of students	Knowledge of the notion circle. Knowledge of natural numbers. *Elementary knowledge of computer work
Expected outcomes	Comprehending the notion of circle and diameter. Intuitive perception of the circumference of a circle. Ability to compare circles based on radius. Adopting the concept of the real number on an intuitive level. Mastered a new tempera painting technique. Practiced reading, understanding, and interpretation of a dramatic text.
Expected duration of the activity	2 school hours, with the possibility of extension to a themed day
Preparation of activities	Downloading the pdf/pptx of the text. Preparation of appropriate equipment for painting. Printing sudoku puzzles. https://inamath.uniri.hr/is-there-something-hidden-in-circles/ *Access to the internet for activities 9 and 10. https://www.britannica.com/biography/Archimedes https://www.britannica.com/biography/Albert-Einstein *https://scratch.mit.edu/help/videos/ * https://microbit.org/get-started/first-steps/introduction/
A detailed description of all teaching activities	**Activity 1: Reading the preparatory text that is projected on the board.** The magical spring number π A small dramatic text with a dash of mathematics *Ana (student of the 4th grade of Elementary School "Jovan Dučić", a wise girl, a big fan of apple pie and painting):* What is that title? Numbers are numbers, there is nothing magical about them. Actually, they are a bit boring... And what about

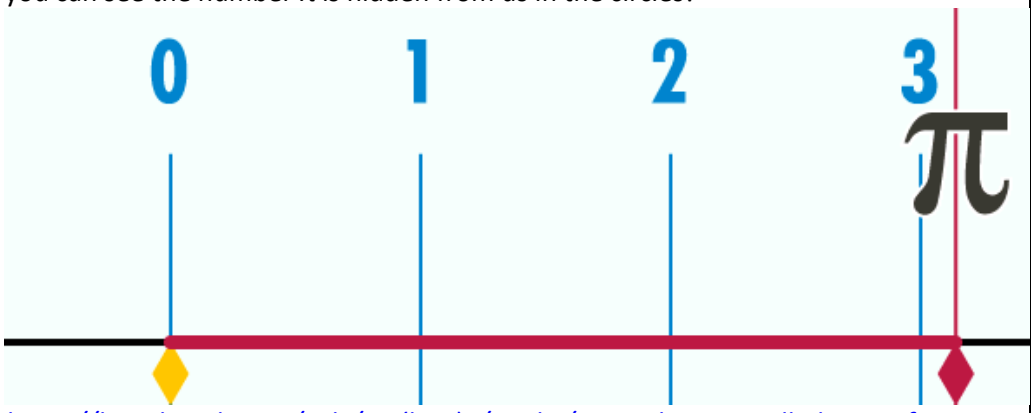
	spring? They bother us with numbers also in autumn and winter. Maybe a little less in the summer, we are saved by the summer holiday. 😊 *Ivana (professor, for some unknown reason a big fan of mathematics):* This number is different. It is very stubborn, and persistent and is recorded in a very strange way. Let's use a fancy Greek letter (raise your hands all gyros fans!) to spell it out π We read that letter as pi. *Ana*: I'm not convinced. And why do we write it with a Greek letter? Although, it's truly nice, we could paint it nicely, with nice brushstrokes. *Ivana:* Of course, we can use digits to write it, but that number is so persistent that it won't stop. It is one of those numbers that is written with a dot. For example, when you read a recipe for an apple pie, and it says 1.5 kg of apples. That's one whole kilogram and another half kilogram. That dot is placed after that whole kilogram and tells us that we need more than a kilogram of apples, but still less than two kilograms. If we want to write π down, we need 3, then a dot after the three, and a lot more digits. Even today, supercomputers are competing to discover more digits of our number. Someone from Switzerland took 108 days last year and found more than 62 billion (who knows how many zeros there are, raise your hand 😊) digits after the comma. And there is no regularity, no repetition, the digits appear as they please. Without a lot of math, we can't know which one is next. Here's how it looks, but let's not overdo it with the numbers after the comma, we have enough for e.g., 50 pieces 3.14159265358979323846264338327950288419716939937510 *Ana:* Well, it is really persistent! Who will remember that? *Ivana:* Yes, it really exaggerates. That's why most people memorize only the first two digits after the dot 3.14 The rest is in books and on the Internet, so if you need it... *Ana:* I understand now! That's why we're talking about this number today! It is 3 14, that's March the fourteenth! But why the spring number? Calendar-wise, spring hasn't started yet, we learn when spring starts, you can't confuse us... Ivana: You are right. But spring is so close, so it didn't make sense to call it a magical winter number… *Ana*: And what is it good for? Really, does it do anything, or do mathematicians have nothing better to do but make up numbers? *Ivana (laughing): Yes, they are very bored, so they make up numbers all day long.* *Ana:* I knew it! Surely that's why we must learn distributive property! I knew it, I knew it, I knew it,... *(Ana runs around the classroom and triumphantly sings "I knew it, I knew it...").* *Ivana:* Stop, stop! I'm joking! Of course, it has a purpose. Let me show you. Do you know what a circle is? *Ana (slightly offended):* What kind of question is that, of course, I know.

Ivana: I'm sorry, I didn't doubt it. Let's draw some circles.

Activity 2: On the floor of the classroom or in the schoolyard, draw three large circles of different diameters with chalk. Three volunteers carefully measure the lengths of the drawn circle lines in steps, i.e., the circumferences of the circles, for all drawn circles. Let's explain the concept of the circumference of a circle. Let's notice the radius and connect the radius with the number of steps. Compare the results and discuss. Drawing circles on paper and a blackboard is also possible, and with the help of a string that we place on the drawn circles, we explain the concept of circumference and compare the results.

Ana: But that's not fair, they all have different shoe numbers! Of course, the number of steps varies even for the same circle!

Ivana: Exactly. The number of steps will depend on the length of our feet. But, to avoid confusion, the exact circumference can be told to us by the number π. If you multiply the diameter of the circle by π, you get the circumference of the circle. Actually because of the circumference of the circle (and the surface, but that's another story) mathematicians began to calculate and discover the number π. As you can see the number π is hidden from us in the circles!



https://hr.wikipedia.org/wiki/Pi_(broj)#/media/Datoteka:Pi-unrolled-720.gif

Ana (laughing): It hid really well.

Ivana: Look at it like this, it is also hidden in the apple pie, the circular one, of course. On one hand, a pie is circular in shape, so it has a circumference that we calculate using π, on another pie is pronounced the same as the letter π.
Ana: I'm hungry now. Good thing lunchtime is near. I hope it's pie on the menu today. 😊
But, why magical? Yes, a pie has a bit of magic, but not enough.

Ivana: If you try hard and look closely at the digits of the number π, you can certainly find the date of your birthday. Sometimes it's immediately noticeable (for example, in the case of my uncle, it's March 14th), and sometimes you need to look at a lot of numbers, but it's definitely there. And you can make it magical too. You said yourself that you could draw him with fine brushstrokes, so let's draw. Maestro, some light spring music please, so let's make it our π magic with tempera.

Activity 3: Drawing the letter π and circles of the different radius using the technique intended for art class work. After class, organize an exhibition of works.

Ivana: Yes Sofia *(Anna's best friend, a bit of a computer freak, knows which graphics card is the best and why you should use a mechanical keyboard, and is also a big fan*

	of apple pies), you can use modern methods. Turn on that computer, you will be in charge of the graphic design. Gather your team. And when you're all done with the drawings, we can move on to the brain teasers. **Activity 4:** Depending on the software available in the school, children draw circles of different dimensions and colors, and arrange them in given shapes (tree, house, flower,...). **Activity 5:** Solving Pi sudoku puzzles. Depending on the affinity of the children, it is possible to divide them into groups and organize competitions. **Activity 6:** A small competition in memorizing the decimals of Pi.
Expanding activities	The possibility of organizing a themed day. In agreement with informatics teachers, older students mentor younger students: **Activity 7:** Making simple animations in Scratch. **Activity 8:** Depending on the availability and affinity of the children, programming the micro:bit car for circular movement. Two toy cars move along different circular paths. Discuss which one returned to the starting point first and why. Connect with the concept of scope. Historical aspects **Activity 9**: Introduce the children to Archimedes. Conjure up to the children the historical period in which he lived, and explain that he was the first to calculate the number Pi. Tell the legend of his death. **Noli turbare circulos meos! – Do not touch my circles!** **Activity 10:** Introduce children to Einstein. Conjure up to children the historical period in which he lived, his most famous achievements. Connect it to the subject through the date of birth.
Additional notes	If a student attends classes according to an individual educational program, it is necessary to include a personal companion in the preparation and choose activities in which it is possible to include the student as much as possible.
Authors	Ivana Štajner-Papuga

Title	**Quadrature of the continent**
Key words	Rectangle, square; Surface area; Measure, unit of measure; Measuring, counting; Continent, country.
Summary	This scenario is raleted to: 1. Mathematics: calculating the area of rectangles and squares 2. Geography (science): planet Earth, the ratio of land and water surfaces, continents. The idea is that pupils by appling knowledge on calculating the area of rectangles and squares, using the computer program "Quadrature of continent", determine the total area of different continets. They can determine the area of Australia and South America, based on the example that the teacher presented for the African continet.
ICT tools included in scenario	Computer program "Kvadratura" (Quadrature) and "Kvadratura kontinenta" (Quadrature of the continent)
Areas	A1: Mathematics A2: Science A3: Mothertounge language A4: Music A5: Art A6*: Informatics A7*: Project class
Topics (for each selected Area)	A1: Rectangle and square – surface area of rectangle and square
	A2: Our country - part of the world. Geographical map. Surface areas of continents and countries
	A3: Direct speech
	A4: Rhythmic accompaniment to the song
	A5: Composition - line, surface, color
	A6*: Work on computer work - Basics of digital literacy
	A7*: Educational project "Lugram"
Expected prior knowledge of pupils	Knowledge of the terms rectangle and square; Knowledge of surface measurement units; Knowledge of natural numbers; *Elementary computer knowledge
Expected outcomes	General outcomes The student will be able to: • determine the surface area of the given figure with a non-standard unit of measure; • read, compare and convert surface measurement units; • calculates the area of squares and rectangles; • solve problem tasks in the context of measurement Operationalized outcomes The student will be able to: • using a square grid, determine the area of the figure with a non-standard unit of measurement; • understands that the area of a rectangle (square) is equal to the product of its width and length; • apply the equation in calculating the surface area of rectangles and squares;

	• calculates one of the sides of the rectangle if the other side and surfeca area of the rectangle are known; • converts measure units for surface area; • solves problem tasks in the context of measurement.
Expected duration of the activity	2 school hours (90 mins in total)
Preparation activities	Preparation and printing of teaching sheets. Downloading and installing computer programs "Quadrature" and "Quadrature of the continent" (www.lugram.net/download, install/unpack the programs on a disk where the user has R/W privileges), information about programs and their use in classes: http://www.lugram.net/kvadratura.html. Preparation of square meter and square decimeter models. Audio material with a song which mentions continets on mothertong language (e.g. We went to Africa, https://www.youtube.com/watch?v=3NmI1HNMjQE, youtube). Preparation of Orff's rhythmic instruments, practice of singing and rhythmic accompaniment of the song. Preparation of computers and projectors in the classroom. Additional material for implementation of this scenario please download from the link - worksheets docx: https://inamath.uniri.hr/wp-content/uploads/2022/12/Worksheets.docx -worksheets pdf: https://inamath.uniri.hr/wp-content/uploads/2022/12/Worksheets.pdf Information about classes: https://www.20oktobarsivac.net/2021/12/20/ugledni-ogledni-cas/, https://www.20oktobarsivac.net/2021/12/21/kvadratura/ (there is a selection of the Cyrillic/Latin alphabet).
Detailed description of activities	First class Introductory part (10 min) Activity 1. Teacher: Today's lesson begins with a song with rhythmic accompaniment by Orff's instruments. The song "We went to Africa" - rhythmic accompaniment with Orff's instruments. Activity 2: Teacher: What is Africa? List the names of the other continents. How are they different? On which continent is our country? Pupils answer the questions. Teacher: Where is pepper planted? What unit of measure do we use to measure the surface area of the field? Pupils answer the questions. Teacher: State the name of the basic unit of measure for area. What are the units for measuring area less than m^2? What units are larger than m^2? How did we measure the area of figures until we knew the units of measure for area? Pupils answer the questions. Main part (25 min) Activity 3: Teacher (story): Pera from Banat (or the part of your the country where you live) traveled to visit his friend in Africa. Simba lives on a large estate with a field behind it. He will plant Pera's pepper there. In order to do this job, they must know the area

of the garden, field, pepper warehouse...
Pupils listen to the story in order to relate its content to the tasks that will do later.

Activity 4:
Giving worksheets and solving tasks (Teaching worksheet no. 1)
1. The rectangular property has a length of 62m and a width of 45m.
Calculate the area of the garden if there is a house that occupies the property area in a shape of square with a width of 14m.
What shape is the property? What shape is the house? What do we know? What should we calculate?
Pupils read the task, answer the questions, solve the task on the worksheets and on the school board.

2. Simba has a rectangular field whose area is 4ha 80a with a lengthof 600m.
a) Calculate the width of the field.
b) How many rows of peppers can be planted in this field if the field can have two rows of peppers per each meter wide?
What is the shape of the field? What do we know? What should we calculate?
Pupils read the task, answer the questions, solve the task on the worksheets and on the school board.

Activity 5:
Handing out worksheets and solving tasks (Teaching worksheet no. 2)

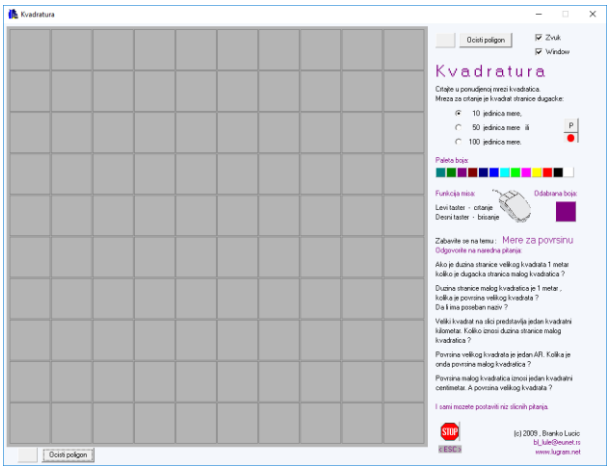
3. Calculate the length of the square warehouse, if its floor area is $81m^2$.
Pupils solve the task completely independently on the worksheets and later on the school board.

Final part (10 min)

Activity 6: (using a computer in the classroom)
Task for pupils:
Represent the area of the warehouse using the computer program "Quadrature".

- What surface area one square represent? (Area of one square meter)
- The teacher directs the students to the following conclusion: We often cannot show everything in natural size. On the geographical map, we show reduced forms of relief. On the settlement plans, we show reduced objects.

By coloring the squares, the pupils represent the given area, answer the questions, make a conclusion.



Second class
Introductory part (10 min)

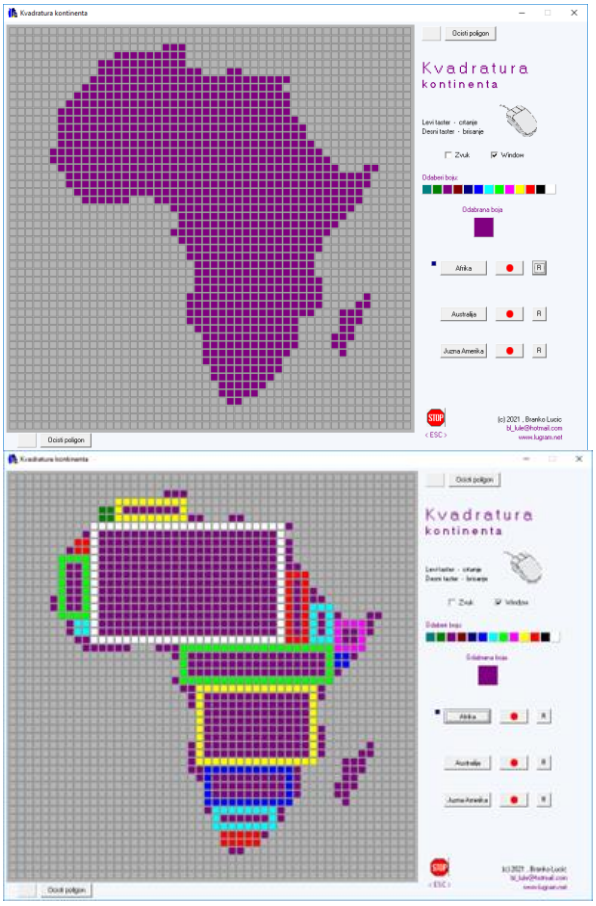
Activity 1:
Conversation: We are still in Africa.

The teacher directs the pupils to observe the "Quadrature of Africa" (computer program " Quadrature of the continent", projection of the content on the canvas/blackboard in the classroom).

teacger presents a problematic situation: "How to determine the number of squares that cover the map of Africa as easily and quickly as possible?"
Pupils are expected to:

- notice the division of the map of Africa into squares,
- they come to na idea of drawing rectangles and squares on the surface of Africa, and then by summing up the surface areas of them all together camo to the solution of the problem,
- connect experiences gained by calculating the area of squares and rectangles and use them in solving the problem.

The teacher using the program "Quadrature of the continent" draws several rectangles and squares on the "Quadrature of Africa", and demonstrates the idea of "quick counting" of the squares covering the map of Africa. It then shows a finished example of the "Quadrature of Africa" where rectangles and squares covering most of the territory of Africa have already been drawn.



Main part (20 min)

	Activity 2: Option 1 (realization in the regular classroom) Handing out teaching sheets and solving the task (Teaching sheet no. 3) Task: Determine the number of squares that covers the map of Australia according to the example of Africa ("Quadrature of Africa"). The teacher walks around the pupils while they are solving the task and provides additional explanations if needed, directs a group of pupils who are solving the task with the "Quadrature of the Continent" computer program, reviews the pupils works and results. Pupils listen to the instructions, noting the similarity with the previously presented example "Quadrature of Africa", and independently (or in pairs) solve the task on the worksheet, while a group of pupils solve the task with the "Quadrature of the Continent" computer program. Afterwards, they read the results and compare them. Option 2 (realization in a classroom equipped with computers) Task: Determine the number of squares that covers the map of Australia according to the example of Africa ("Quadrature of Africa"). The teacher walks around the pupils while they solve the task and provides additional explanations if needed, reviews the students' work and results. Pupils listen to the instructions, noting the similarity with the previously presented example "Quadrature of Africa", and independently (or in pairs) solve the task with the "Quadrature of the Continent" computer program. Afterwards, they read the results and compare them. Note: the program "Quadrature of the Continent" (in the current version) does not have a teacher's and student's mode of use. There are action buttons that offer information on how many squares cover the surface of the continent. The result of the student's work must be supported by an appropriate procedure that is shown graphically and computationally. That's why the mentioned working buttons are not hidden. If practice shows that it is necessary, a "key" can be introduced that will be available only to the teacher. Final part (15 min) Activity 3: The teacher sets the task: Write, using directed speech, what Pera and Simba said to each other at the end. Students write sentences and then read some examples. Discussion with students: What was interesting to you in this lesson? What was difficult for you? What part of the lesson did you like the most? Homework: 1. Using the program "Quadrature of the continent" (or teaching sheet no. 4) determine the number of squares covering the map of South America. 2. Solve the tasks from the teaching sheet no. 5.
Opportunities to expand activities	Within the Informatics class: Area of cuboid and cube. As part of the educational project "Lugram", also in the same class (Project class), creating the components of the Lugram puzzle.
Additional notes	/
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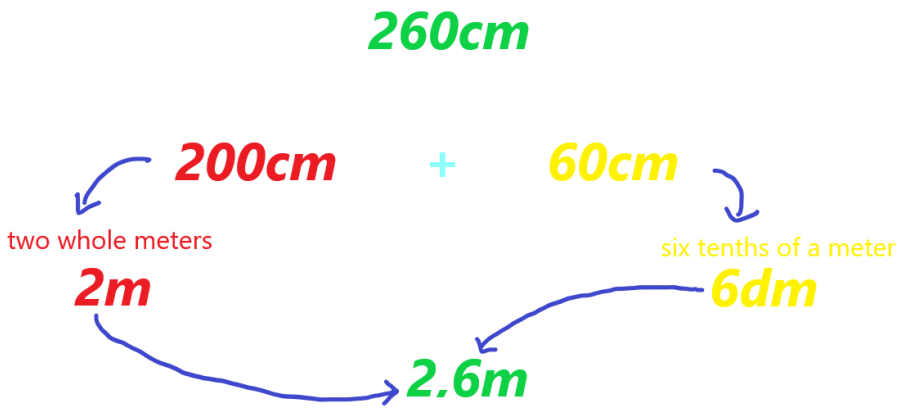
Title	**Lazy dot**
Keywords	Natural numbers up to 1000 Measure for length Decimal notation
Short description	Connecting physical education, arithmetic operations for numbers up to 1000, the measurement for length, and decimal notation. The student's height, the length of the long jump, and the length of the vortex flight are recorded, the expression of length/height in both centimeters and meters is practiced using decimal notation, and addition and subtraction of the values recorded in this way are practiced.
ICT tools included	Drawing software (Paint) Scratch
Areas (select)	A1: Mathematics A2: Physical education A3*: Computer science A4*: Art *possibility of expanding activities
Topics (for each selected area)	A1: decimal notation, addition, and subtraction
	A2: long jump
	A3*: Introduction to the drawing software and Scratch
	A4*: Getting to know certain painting techniques
Expected prior knowledge of students	Knowledge of basic arithmetic operations with natural numbers up to 1000. Knowledge of length measurement *Elementary knowledge of computer work
Expected outcomes	Understanding decimal notation and the procedure for adding and subtracting numbers in decimal notation
Expected duration of the activity	2 school hours, with the possibility of extending it to a sports and art day
Preparation of activities	Download a pdf/pptx of the introductory text. Download the pdf of the graphics for activities 2. Download the pdf for activity 5. Download the pdf for activities 7. Download the pdf for activity 8. https://inamath.uniri.hr/lazy-dot/ *https://scratch.mit.edu/help/videos/
A detailed description of all teaching activities	**Activity 1: Reading the preparatory text that is projected on the board.** The physical education class is in progress, the children are warming up by throwing the ball into the basket, and the twins Anđa and Ines are arguing. First quietly, then louder and louder. *Anđa (throws a ball that bounces off the hoop and hits Ines):* Well, it's easy for you, you're 137 tall, and I'm only 1.37. Of course, it's easier for you to get the ball through the hoop! *Ines (doesn't even look at her sister, but nonchalantly picks up the ball and it goes effortlessly through the hoop):* What are you talking about?! We are of the same height, only you are a lost case when it comes to basketball! *Anđa (now very angry):* Well, no, you have the full 137, and I barely have 1 and a little more, you know very well that the doctor at our checkup dictated that to a nurse...

	Teacher Vlada has been watching this exchange of sparks from a safe distance for some time, but now he simply must intervene. *Vlada*: Just a moment you two, how tall are you? *Anđa and Ines (at the same time):* 137 centimeters/1.37 meters! *Vlada:* Yes, I see where the problem is. You two were sleeping during the lesson on how we measure length! *Anđa and Ines (this time in unison):* We weren't, we really weren't! We measure the length in meters, decimeters, centimeters, and millimeters. But meters are the main thing! *Anđa:* But what does our height have to do with length... wait a minute, yes, our height is actually just length going up! *Ines*: That's right! And your 1 and something else is in meters, and my 137 are centimeters! That's what that doctor said! *Anđa:* But I still don't understand. Well, 100cm is 1m, and 10dm is the same as 1m, we learned that in class (see, we were not sleeping during the lesson), but what should we do with 137cm? *Vlada:* Excellent question! It's just in time for math class, so we can clear that up. *Anđa and Ines (again in unison):* No, not mathematics! It's really nice for us here in the schoolyard, at PE class. **Activity 2: Group discussion of the problem that bothers Anđa and Ines, and Vlada's explanation that follows.** *Vlada:* Don't worry, we're staying outside. Bear with me for a moment while I explain what **decimal notation** is. As you said, when we talk about length, meters are important and it's really convenient when something is one meter or 3 meters, or 15 meters long. But, you must have noticed, it is not always like that. Look at this basketball hoop. It is at a height (as Anđa nicely explained, the length that goes up) that is greater than 2m and less than 3m. We have 2 meters and another piece that is less than a meter. If what is written on the basketball hoop is correct, we have 60 cm more. When we write down our number, we separate that part from whole meters with a dot. This dot is called a **decimal point**, and writing a number down with this decimal point, we get a decimal record. Now, for our basketball hoop, the part with the whole number of meters is easy, but we need to write down precisely the part that we have left. Here's an example. Vlada draws with chalk on the ground. Fortunately, rain is forecast for the afternoon, so everything will wash away, and the headmistress of the school will not scold them.



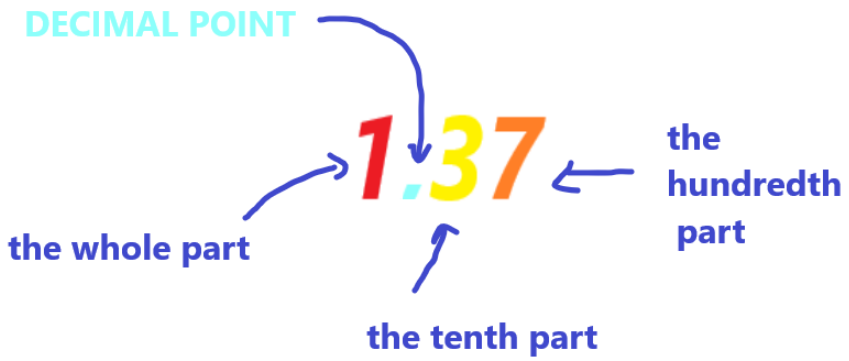
Ines: Okay, I got this. But that example had a nice 6 tenths (here, I listened in the last class, I know that when we divide a meter into ten equal parts, each part is a decimeter), but what should we do with my extra 37 cm?

Vlada: You made an excellent point. In that 37 cm, you have 3 complete decimeters and another 7 centimeters extra.

Anđa (excitedly interrupts Vlada): I listened in class too! A centimeter is a hundredth part of our meter, so we have 3 tenths and another 7 hundredths, and that's how my height turns out to be 1.37!

Vlada: Exactly! The first digit after the decimal point tells us how many tenths of a whole, we have, and the second digit tells how many hundredths are left. Let's analyze your height:

Vlada: Let's repeat what is what, and it's time to play.



Activity 3 – let's go to the schoolyard:
Children are divided into 5 groups. Each group is in its own part of the yard and gets colored chalks. The task is writing down their height in decimal form, following the previous example. The group that finishes first has the right to choose which ball game they play after discussing all the entries. In case of bad weather or crowds in the schoolyard, each group gets an A3 paper and crayons and makes as colorful a record of their height in decimal form as possible.

Activity 4 – playing the ball game chosen by the winners from the previous activity, to shake things up a bit. 😊

Activity 5a - we continue with the text prepared on cards, very convenient for reading in the schoolyard:

Ines (raises her hand, all jumpy with impatience): Teacher, teacher, I must ask something. If it says 2.3 and 2.30, is that the same? It bothers me a bit, 30 is greater than 3, but that zero would be the number of hundredths, so it still seems the same to me...

Vlada: You made an excellent point. That zero is the number of hundredths, and there are no hundredths in your example. It is quite enough to write down only 2.3. Well done!

Andja (she is also jumping, there are many questions): If it says 3.04, it means that there are no tenths, but we have 4 hundredths, right? Now we must write that zero because without it this would be 3.4. That's three whole parts and 4 tenths and it's not the same!

Vlada (very pleased): You are absolutely right! Let's practice it a little, with long jumps. Today we are scheduled to do the long jump in PE class, but this class of mine apparently jumped out a bit...

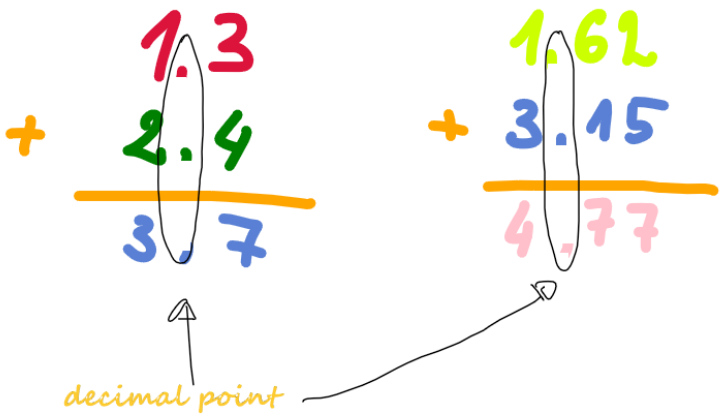
Activity 6 - Leaping Decimals:
Children are put into pairs. One student does the long jump, another helps him measure the distance in centimeters, then together they translate it into decimal notation. The second student jumps and the first student helps him measure the distance in meters, and together they translate it into centimeters.

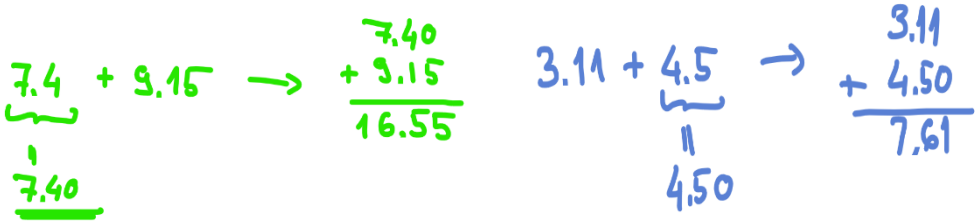
Activity 5b – some more cards:
Andja: And what if I jump, then jump again and again? How far did I jump?

Vlada: We can measure each of your jumps, then add up the lengths.

Ines (concerned): That's easy if we measured everything in centimeters. What if we had numbers in that decimal notation? That dot bothers me as if it is also jumping.

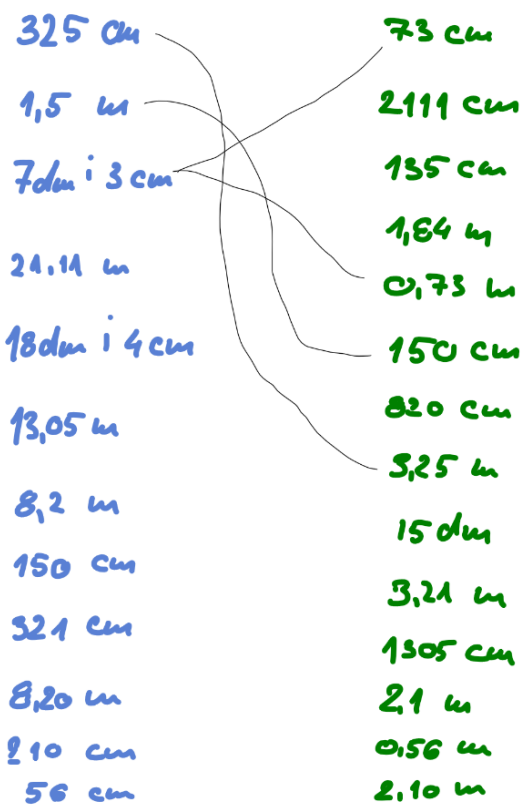
Vlada: Don't worry. That dot, the decimal point, is very lazy, it doesn't move anywhere. We can subtract and add, but we must make sure that the dots line up. And carefully when you add those that have hundredths with those that only have tenths. Here it is:





Activity 7 - Lazy Dot: The children are again divided into pairs. Now they jump three linked jumps. One student jumps and the other helps him measure all three jumps in centimeters, then they switch. Jump lengths are marked with s1, s2, and s3. Children fill in the following table

	in centimeters	decimal notation, in meters
S1		
S2		
S3		
S1+S2		
S1+S3		
S2+S3		
S1+S2+S3		
3m-S3		
4,2m-S2		
6,12m-(S1+S2)		
6,12m-S1+S2		

	There must be emphasized to be especially careful with the last two items. The first couple to do it correctly can divide their friends into two teams for the next game. **Activity 8 – quiz – back to the classroom:** Two sets of lengths have been written on the blackboard. Each team chooses a representative who, with the help of all team members, connects the same values. Loud cheering is allowed. 😊 Each correct connection brings 2 points, and for each wrong connection, a point is deducted. Time is limited to 5 minutes and the team with more points wins. The winning team chooses math homework. 
Expanding activities	The possibility of organizing a themed day: In agreement with informatics teachers, older students mentor younger students: **Activity 9:** Depending on the drawing software available in the school, we illustrate activity 3. **Activity 10:** Making simple scratch animations to illustrate activities 6 and 7. **Activity 11:** With the teacher's supervision, search the Internet to find interesting facts about the long jump. Research and who in the animal world jumps well. **Activity 12:** We illustrate the interesting things discovered during the previous activity with different painting techniques that were done within the art classes.
Additional notes	If a student attends classes according to an individual educational program, it is necessary to include a personal companion in the preparation and choose activities in which it is possible to include the student as much as possible.
Authors	Ivana Štajner-Papuga