

Threshold Concepts in Mathematics

SciMaG – Work Package 2

Threshold concepts are conceptual gateways that can open a previously inaccessible way of thinking about a subject and enable further learning. They are commonly described as potentially transformative, irreversible, integrative, bounded, and troublesome. Not every difficult or important concept is necessarily a threshold concept; the key issue is whether understanding it changes the learner's way of seeing and using disciplinary knowledge.

Identified Threshold Concepts

Fractions as a Threshold Concept

Fractions mark a transition from whole-number reasoning to rational-number reasoning and support later proportional, algebraic, and measurement thinking.

Threshold characteristic	Explanation
Troublesome	Students often apply whole-number reasoning to fractions and struggle with magnitude, equivalence, and operations.
Transformative	Understanding fractions changes how learners conceptualise number, magnitude, quantity, and relationships between quantities.
Integrative	Fractions connect whole numbers, decimals, percentages, ratios, proportional reasoning, measurement, and algebra.
Bounded	Fractions belong to rational-number reasoning and require learners to distinguish their properties from those of whole numbers.
Irreversible	Once conceptually understood, fractions become a flexible representation of quantity rather than merely a set of procedures.

Related SciMaG learning activity: Let's Play with Fractions – S48

Spatial Visualization as a Threshold Concept

Spatial visualization is essential for understanding geometry. It enables students to mentally manipulate objects, interpret different perspectives, and connect three-dimensional objects with two-dimensional representations.

Threshold characteristic	Explanation
Troublesome	Students often find it difficult to mentally rotate or transform objects and interpret different views of the same structure.

Transformative	Once developed, spatial visualization changes how students approach geometry by enabling them to mentally manipulate and anticipate spatial relationships.
Integrative	It connects geometry, measurement, transformations, coordinate systems, and graphical representations, and supports learning in STEM fields.
Bounded	The concept is primarily situated within geometry and spatial reasoning, where students learn to distinguish between objects, representations, and perspectives.
Irreversible	Once developed, spatial visualization becomes a lasting strategy that students continue to use when solving spatial and geometric problems.

Related SciMaG learning activity: Build and Reveal – S50.

Mathematical Modelling as a Threshold Concept

Mathematical modelling involves translating real-world situations, words, or phenomena into mathematical representations and using mathematics to solve problems. It develops progressively throughout mathematics education.

Threshold characteristic	Explanation
Troublesome	Students often have difficulty identifying the relevant information in a problem and translating it into an appropriate mathematical representation.
Transformative	Understanding modelling shifts students from memorising procedures to using mathematics to represent, analyse, and solve unfamiliar problems.
Integrative	It connects different areas of mathematics and provides a bridge between mathematics and subjects such as physics, chemistry, computing, and engineering.
Bounded	Students who have not mastered modelling tend to rely on familiar problem patterns and can become confused when the wording or context changes.
Irreversible	Once students understand how to formulate mathematical models, they can apply this approach to increasingly complex problems across different contexts.

Related SciMaG learning activities: S16, S17, S18, S19, S27, S31, S32

Vectors as a Threshold Concept

A vector represents a directed quantity with magnitude and direction. The concept provides an important foundation for geometry, linear algebra, physics, and other technical fields.

Threshold characteristic	Explanation
Troublesome	Students often struggle to understand the abstract meaning of a vector and tend to focus on formulas and procedures.
Transformative	Understanding vectors enables students to approach geometric and spatial problems at a higher level of abstraction.
Integrative	Vectors connect geometry, algebra, physics, motion, forces, direction, parallelism, and orthogonality.
Bounded	Misunderstanding the definition of a vector often makes its operations and geometric interpretation difficult to understand.
Irreversible	Once students understand the meaning of vectors, they can apply vector properties and operations across different mathematical and scientific contexts.

Related SciMaG learning activities: S28: Sudoku with Vectors in the Coordinate System for Pairs; S29: Team Sudoku with Vectors

Functions (and bijections) as a Threshold Concept

A function describes a relationship in which each element of one set is associated with exactly one element of another set. Functions and bijections are fundamental concepts that underpin many areas of mathematics.

Threshold characteristic	Explanation
Troublesome	Students often find the abstract definitions of functions, domain, codomain, injection, and surjection difficult to understand and apply.
Transformative	Understanding functions enables students to recognise and analyse relationships between variables and provides a foundation for mathematical modelling.
Integrative	Functions connect graphs, proportionality, inverse functions, probability, modelling, and many areas of mathematics and science.
Bounded	Students may know definitions and procedures but still confuse concepts such as domain and codomain or treat injectivity and surjectivity mechanically.
Irreversible	Once understood, students can explain functions in their own words and apply their properties correctly in new situations.

Related SciMaG learning activity: S20: Let's Play with Functions

Measurement as a Threshold Concept

Measurement involves understanding what is being measured, how it is measured, and which unit is used. It is a foundational concept developed throughout mathematics education.

Threshold characteristic	Explanation
Troublesome	Students may memorise units and formulas but still struggle to understand what is being measured and how different quantities are related.
Transformative	Understanding measurement provides a foundation for more advanced mathematical concepts such as area, volume, and measure, as well as for measurement in science.
Integrative	Measurement connects geometry, arithmetic, algebra, calculus, and scientific disciplines such as physics and chemistry.
Bounded	Students may solve routine measurement problems while still making conceptual errors in unit conversion, formula selection, or interpreting word problems.
Irreversible	Once the concept of measurement is understood, students can apply it flexibly, understand derived units, convert measurements, and solve increasingly complex problems.

Related SciMaG learning activities: S36: Through Puzzles to Volume; S37: Through Transformations to a Unique Puzzle

References

Meyer, J. H. F., & Land, R. (2005). Threshold concepts and troublesome knowledge (2): Epistemological considerations and a conceptual framework for teaching and learning. *Higher Education*, 49(3), 373–388. <https://doi.org/10.1007/s10734-004-6779-5>